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TWO-QUASI-PARTICLE STATES IN EVEN-MASS NUCLEI WITH DEFORMED EQUILIBRIUM SHAPE

BY

C. J. GALLAGHER, JR., AND V. G. SOLOVIEV

København 1962
i kommission hos Ejnar Munksgaard

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Synopsis

The energy levels in even-even nuclei and the beta transition rates between even-even and odd-odd nuclei in the $150 < A < 190$ region are analyzed. The excitations in these nuclei are describable as rotational states based upon intrinsic states. With the exception of systematically occurring excited $2+$ states in even-even nuclei, it is shown that the intrinsic states are describable as two-particle excitations, if the Nilsson wave functions are used to characterize the single-particle states. The level energies and beta transition rates are compared with theoretical values which take into account the effect of pairing correlations. The pairing correlations are evaluated with inclusion of "blocking" as described in a previous paper by one of the authors (Mat. Fys. Skr. Dan. Vid. Selsk. **1**, 11 (1961)).

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I. INTRODUCTION

Within the last few years significant theoretical clarification of the coupling schemes applying to the low lying states of nuclei has taken place[1,2,3,4].

In the mass region $150 < A < 190$, where the strong coupling model of Bohr and Mottelson is a useful description of the nuclear system[4], an extensive classification of the properties of intrinsic nuclear states has been made possible through the introduction by Nilsson of single-particle states calculated for a deformed potential well[5]. In the compilation of Mottelson and Nilsson it has been clearly demonstrated that many of the properties of the intrinsic levels of odd-mass nuclei in this region can be described by these wave functions[6]. The ground states of odd-odd nuclei in this region are now known to be simple proton-neutron systems in which the last odd proton and odd neutron are usually the Nilsson states appropriate for the Z and N in question, coupled in such a way that the particle intrinsic spins couple parallel[7]*. Finally, on the basis of an analysis of beta decay rates from deformed odd-odd nuclei using simple two-particle Nilsson wave functions and single-particle operators, it has been pointed out that there exist some experimental data which indicate that the excited levels of deformed even-even nuclei may also be describable simply as two-particle Nilsson proton or neutron states strongly coupled to the deformed core[8].

We can summarize these results for deformed odd-odd and odd-mass nuclei by the statement that the experimental data indicate that, at least at low excitation energies, unpaired particles in these nuclei, while being strongly coupled to the deformed core, apparently interact only weakly, if at all, with each other. The extent to which this is also true in the excited states of even-even nuclei has until now not been determined.

A salient feature of the excitation spectra of all deformed even-even nuclei observed to date, and one which has not been explained by the above-mentioned

* In some cases, the experimental data indicate that configurations other than the last odd-proton odd-neutron state are the lowest. The extent to which this competition between different configurations occurs is not known experimentally at present. However, even where the observed proton-neutron configurations differ from those expected, the lowest lying state is that in which the intrinsic particle spins are coupled parallel.

models, is that an energy gap of approximately 1 Mev is observed between the ground and first-excited intrinsic states. BOHR, MOTTELSON, and PINES pointed out that this gap might arise, in analogy to the energy gap in super-conductors, from correlations between coupled particle pairs in the core[9]. Detailed mathematical studies of this problem show that a general feature of the excitation spectra of even-even nuclei based on pairing correlation calculations is the occurrence of an energy gap, the exact energy of this gap depending on the more detailed assumptions within the theory[10,11]. We will discuss some of these studies in more detail in Section II. However, in a nuclear model including pairing correlations developed by one of us (VGS), which should be valid for deformed nuclei, certain mathematical difficulties inherent in earlier approaches are avoided. The independence of the quasi-particle excitations in both the odd- and even-mass nuclei, an effect which we noted as an empirical fact above, is also contained quite naturally in this model[12]. In addition, the energy spectra and beta decay transition rates in these nuclei can be calculated on the basis of the model. These features will also be discussed in more detail in Section II.

The level spectra of deformed even-even nuclei are particularly good cases for testing this model, because all the parameters necessary for the calculation of even-even spectra can be adjusted using the experimental data on the levels of odd-mass nuclei and pairing energies from experimental mass tables. Unfortunately, from the point of view of making such a test, the amount of experimental data available on the intrinsic levels of deformed even-even nuclei is at present small. Furthermore, although some evidence has been presented suggesting the presence of two-particle excitations in even-even nuclei[8], a detailed analysis of the existing data which demonstrates that the body of experimental data supports such an interpretation has not yet been presented.

The present paper has therefore been prepared with a twofold purpose in mind. It is an attempt, first of all, to analyze the existing experimental data on the intrinsic levels of deformed even-even nuclei in the $150 < A < 190$ mass region in order to determine whether they can be explained as simple two-particle-like excitations, and secondly, to compare the observed spectra with those calculated on the basis of a specific nuclear model including pairing correlations, with the hope that the comparison will provide a means of determining the degree of validity of the model and hence serve to guide further developments.

In carrying out this comparison, we have borne in mind that many data exist on excitations in even-even nuclei, which, because of their measured transition probabilities and energies, have come to be interpreted as being collective in nature. These excitations, for which no provision has been made in the present model, have been widely discussed in the literature and have been interpreted qualitatively as collective nuclear vibrations[13,14], and much progress has been made in accounting for them in a quantitative way[15]. By defining the intrinsic excitations in the nuclei where these excitations occur we hope that a clearer definition of the collective states will emerge, especially with respect to intrinsic excitations of the same spin and parity.

In Section II the general features of the model are discussed. Section III contains the presentation of the experimental data and their analyses. In Section IV we discuss the data and present the conclusions that we believe can be drawn on the basis of the analysis.

II. THEORETICAL

A. General Features of Pairing Correlations Applied to Deformed Nuclei

Investigations of nucleon pairing correlations in heavy nuclei provide an explanation of those nuclear properties which could not be accounted for within the framework of the independent-particle model, particularly the moments of inertia of deformed nuclei and the gap in the excited spectra of even-even nuclei(10,11,16,17). These investigations have shown that the residual short-range forces between nucleons are attractive, and the ground state of any nucleus is the superfluid state, that is, the state in which the amplitude of the last few pairs is spread over several levels. This state is energetically more favourable than that with successively filled levels of the average field in the independent-particle model.

In the present paper, we attempt to compare some of the observed properties of strongly deformed even-mass nuclei with the results calculated on the basis of a model which applies pairing correlations to nuclei with a deformed core.

This model, in addition to including the average field of the independent-particle model, also includes the short range part of nucleon-nucleon interactions which lead to pairing correlations. These residual interactions between nucleons are described by the Bardeen-type Hamiltonian. The main equations of the problem have been found with the aid of the variational principle and are given in ref. 18.

Basically, the present model is a model of independent quasi-particles. However, in addition, we take into account in a systematic way the influence of unpaired particles on the superfluid properties of the system, i. e., the blocking effect which is important in the case of deformed nuclei.

B. Comparison with Earlier Results

The most important differences between the basic assumptions of the present model and those of previous investigations of the effect of pairing correlations(10,11) are:

a) it takes into account the influence of quasi-particles on the superfluid properties of the nucleus, i. e., the blocking effect, and therefore differences exist between the properties of the ground and excited states in the different models;

b) in it the number of particles is on the average equal to the true number of particles in the nucleus for both the ground and excited states.

The concept of independent quasi-particles inherent in the present model seems to find some justification in experimental data. Thus, the clear correspondence between the levels predicted by the Nilsson diagram and the observed level spectra in odd-A nuclei in the region of deformed nuclei seems to indicate that the Nilsson states are good representations of independent quasi-particles. The two-Nilsson-particle interpretation of the energy levels of deformed even-mass nuclei, which has been used by one of us (CJG) to analyze the beta-decay rates of deformed odd-odd nuclei and the levels of deformed even-even nuclei, is also essentially an independent quasi-particle model.

It should be noted at this point that the ideas discussed here for the $150 < A < 190$ mass region should be equally valid in other mass regions in which the nuclear core is deformed.

C. Calculations

In the present paper, we present the results of calculations of the spectra of two-quasi-particle states in even-even nuclei and corrections to beta-decay transition rates arising from nuclear superfluidity.

The main assumptions underlying the detailed calculation of the spectra of two-quasi-particle levels in even-even nuclei and the magnitudes of beta decay log ft's discussed below are formulated in order to exclude as much as possible the ambiguity and inaccuracy associated with the poor knowledge of the levels and wave functions of the average field. Thus, instead of attempting to calculate the even-even spectra and log ft's directly from the wave functions and energies as given by Nilsson, the Nilsson potential was first adjusted to give the best possible fit with the observed odd-A level spectra and experimental pairing energies, and then the calculations were carried out using the adjusted values.

1. *Energy Levels of Deformed Even-Even Nuclei*

We have investigated the properties of the strongly deformed nuclei in the region $150 < A < 190$. The nuclei under consideration were divided into three groups: the first group with $156 \leqslant A \leqslant 174$ $(64 \leqslant Z \leqslant 70;\ 92 \leqslant N \leqslant 104)$, the second group with $174 < A \leqslant 186$ $(70 < Z \leqslant 74;\ 104 < N \leqslant 112)$. The nuclei with $A = 152$ and 154, and the levels in the even-even osmium isotopes and W^{184} and W^{186} make up the third group. At the mass numbers where these nuclei occur the deformations are changing rapidly; hence, although they should probably be considered as strongly deformed nuclei, a reasonably correct calculation would require detailed consideration of each nucleus separately. Because such detailed calculations are not in the spirit of the present paper, no calculations have been made for these nuclei.

For the first two groups one set of the single-particle levels of the average field and one pairing interaction constant for both the proton and neutron systems were

Table II 1. *Single-particle energy levels.*

Neutron system				Proton system			
N	Assigned orbital	Energy I	$(\hbar\mathring{\omega}_0)$ II	Z	Assigned orbital	Energy I	$(\hbar\mathring{\omega}_0)$ II
93	$3/2-[521\uparrow]$	0	0	63	$5/2+[413\downarrow]$	0	0
95	$5/2+[642\uparrow]$	0.04	0.04	65	$3/2+[411\uparrow]$	0.04	0.04
97	$5/2-[523\downarrow]$	0.07	0.07	67	$7/2-[523\uparrow]$	0.12	0.12
99	$7/2+[633\uparrow]$	0.22	0.22	69	$1/2+[411\downarrow]$	0.20	0.20
101	$1/2-[521\downarrow]$	0.24	0.24	71	$9/2-[514\uparrow]$	0.42	0.30
103	$5/2-[512\uparrow]$	0.29	0.29	73	$7/2+[404\downarrow]$	0.31	0.31
105	$7/2-[514\downarrow]$	0.41	0.33	75	$5/2+[402\uparrow]$	0.36	0.43
107	$9/2+[624\uparrow]$	0.48	0.41	77	$1/2+[400\uparrow]$	0.52	0.52
109	$1/2-[510\uparrow]$	–	0.48				
111	$3/2-[512\downarrow]$	–	0.55				
113	$7/2-[503\uparrow]$	–	0.59				

However, it should be noted that inherent in the present approach is the possibility of investigating the change of the average field in passing from one nucleus to another, as well as investigating the importance of other factors which were not taken into account in the present calculation, above all the interactions of quasi-particles.

The calculations of the basic superfluid properties of the ground and two-quasi-particle excited states were carried out on an electronic computer, using the values of the pairing interactions G_N and G_Z and the schemes of single-particle levels given in Table II 1. The absolute values of the excitation energies of the two-quasi-particle states are calculated on the average with an accuracy of $\approx 10\,^0/_0$, but sometimes are probably not more accurate than $20\,^0/_0$. The calculated energy spectra for the individual even-even nuclei are written down in Section III in the second table for each mass number.

2. *Beta Decay*

a) *Selection Rules for Beta Decay.* Selection rules for single-particle transitions in even-mass nuclei based on Alaga's selection rules for odd-mass nuclei have been given in ref. 8. The main difference between the even- and odd-mass systems was therein shown to be the possibility of Λ-forbiddenness in even-mass nuclei. An additional classification of beta transitions within the framework of the present model was formulated in ref. 19, where its importance from the point of view of the properties of the nuclear superfluid is discussed. This classification is based on the change in the number of quasi-particles during a beta transition, viz., all beta transitions were divided into three groups:

chosen, which gave the best agreement with the single-particle spectra of the odd nuclei and the pairing energies. In Table II 1 are indicated the relative energies of some of the most important levels for the first and second groups of nuclei. Note that the behaviour of some levels of the first group is different from that of the corresponding levels of the second group, which is not unreasonable because of the different deformations necessary to reproduce the observed odd-A spectra.

In this calculation, in order to correct for the observed variation of pairing energies from nucleus to nucleus, a rather rough averaging has been made. After the parameters of the Nilsson potential had been adjusted to give the best fit, we obtained for the whole region $156 \leqslant A \leqslant 188$ the following values of the pairing interaction constant:

$$G_N = 26/A \text{ MeV}; \quad G_Z = 28/A \text{ MeV}. \tag{1}$$

The calculations, when carried out using the accepted scheme of the average field single-particle levels and by the chosen values of G, lead, as shown in ref. 18, to a fairly acceptable description of the behaviour of the single-particle levels of the odd-A nuclei.

An exception is the change in the sequence of the $7/2+[404\downarrow]$ and $9/2-[514\uparrow]$ proton levels, and the change in the spin of the ground state of the odd-N nuclei when $N = 95$. When $N = 95$ the scheme used fails to give correct values for the spin of the ground state of some odd nuclei. Therefore, for the case with $N = 91$, we substituted the level $11/2-[505\uparrow]$ in place of the level $5/2-[503\downarrow]$ which, with the other two levels, drops down a place. We expect this will only affect the neutron levels in *Gd*.

We point out here that, for the given system of levels of the average field and for the G_N and G_Z of fixed magnitude, the calculations based on the present model are entirely unambiguous. That is, because only one configuration of the single-particle levels is used for calculating the properties of a group of nuclei, the results of the calculation cannot be interpreted as a "fitting" of the calculated spectra to the corresponding experimental data. Furthermore, no new parameters have been introduced for the calculation of the properties of the even-even nuclei. Therefore, a comparison of the calculated properties of even-even nuclei with the experimental data is very important from the point of view of verifying whether the present model gives a valid approximation to the effective forces in these nuclei.

A calculation using only two sets of the average field levels and only two values of G for a large group of nuclei is a rather rough approximation, as the behaviour of the average field levels, the equilibrium deformations, and the pairing energies change noticeably. This approximation is made first of all to exclude any arbitrariness in calculating the spectra of even-even nuclei, and secondly to allow comparison of the log ft's in even-A nuclei to those in odd-A nuclei. A third, more practical, reason for so few adjustments is that, although there are relatively few single-particle levels in odd-A nuclei, relatively little is known about them, and the available experimental data did not indicate the necessity of forming more than two groups.

$$\begin{array}{lll} \text{I.} & R(G=0)=1 & 0<R(G\neq 0)<1 \\ \text{II.} & R(G=0)=0 & 0<R(G\neq 0)<1 \\ \text{III.} & R(G=0)=0 & R(G\neq 0)=0. \end{array}$$

The first and second groups contain those beta decays in which only one quasi-particle in the proton (neutron) systems disappears or appears and the configuration of the remaining particles is left unaltered. These are essentially the cases considered in ref. 8. In the third group, which is totally forbidden on the basis of the present model and denoted in the classification of beta transitions by F, are included

a) transitions in which, besides the change in the number of quasi-particles by one, the configuration of other quasi-particles changes (the "non-overlap" forbiddenness of ref. 8);

b) transitions in which the number of quasi-particles of the proton (neutron) system changes by more than one.

Transitions of this latter group imply quasi-particle interactions which are not included in the present independent quasi-particle model and are therefore a powerful means of testing the validity of this important assumption of the model.

b) *Corrections to Beta Decay Transition Rates Arising from Nuclear Superfluidity.* As has been mentioned previously, the effect of the pairing correlation is to spread the amplitude of the last few pairs over a number of states. This distribution will differ if the proton or neutron core in question contains an even or odd number of particles. Thus, for example, a beta transition which changes an odd-neutron core plus an odd-proton core into an even-neutron core plus an even-proton core will involve considerable rearrangement of the many-body system. The effects of these changes on the single-particle transition rate can be calculated on the basis of the present model. As is shown in ref. 20, the superfluid corrections $R = R_Z R_N$ to the beta transition probabilities can be important. (In the present discussion, the notation of ref. 20 is used). Values of $\log(ft)_c$ for even-even nuclei were calculated using experimental data and superfluid corrections determined for the same single-particle beta transitions in odd-A nuclei. The matrix elements were determined from the odd-particle transitions indicated, and the values of the $\log(ft)_c$ listed for the even-mass nuclei were calculated assuming these matrix elements and the superfluid corrections calculated on the basis of the model. The comparison of the beta transition probabilities is valid only when the transition is a non-Λ-forbidden transition of group I or II, in which ΔI is the same in the odd- and even-mass nucleus. We have also calculated $\log[(ft)_e R_\eta]$ which is designed to improve the relative agreement of transitions in even-mass nuclei by correcting the experimental $\log(ft)_e$ for the superfluid (R) and statistical (η) factors.

3. *Other Physical Properties*

Within the framework of the present model it is also possible to consider gamma-ray transitions, magnetic moments, and moments of inertia. Superfluid corrections to gamma-ray transition probabilities are complicated, and together with the fluctuations in gamma-ray transition rates arising from small variations of the nuclear wave functions their role is not at present clear. To the extent that two quasi-particle states are approximated by two Nilsson particles the Nilsson wave functions should be useful for calculating their magnetic moments. Expressions for the magnetic moments of deformed odd-odd nuclei have been given previously[6,7,21] and satisfactory agreement between the observed and calculated moments has been found. The calculation of moments of inertia for the ground states of even-even nuclei has been the subject of several recent papers, and the extension of these calculations to two-quasi-particle excitations in even-even nuclei promises to be a formidable task. In the present paper we shall not attempt a further discussion of these topics.

Several features, especially quasi-particle interactions and other residual interactions, cannot be investigated within the framework of the present model without the addition of perturbation terms in the Hamiltonian. For example, the spin-dependent forces which might be expected to remove the degeneracy of the $\Omega_1 \pm \Omega_2$ doublets in the excited states of even-even nuclei, in analogy to the spin splitting observed in odd-odd nuclei, are not included. At present, however, the role of these interactions is not clear, and we hope that the present investigation will help to clarify it.

As was mentioned in the Introduction, no provision for collective vibrations has been made in the present model. However, the important role of such excitations is experimentally established, and in cases where definite disagreement between calculated and observed energies occurs for states with $K\pi = 0\pm$, and $K\pi = 2+$ we have tended to assign the states as collective in the interpretation. In these cases, we have in general also included the most probable alternative two-quasi-particle interpretation of the states.

III. INTERPRETATION OF THE OBSERVED LEVEL SPECTRA

A. Summary of Parameters Used in Discussing the Spectra of Non-Spherical Even-Mass Nuclei

The analysis of even-even spectra attempted in the present paper is based primarily on the idea that the energy levels of deformed nuclei can be divided into rotational and intrinsic states, because the introduction of pairing correlations into the discussion of the intrinsic structure does not change this description. We will therefore follow rather closely the format used by Mottelson and Nilsson in their extensive analysis of the levels in deformed odd-mass nuclei. Thus, before beginning a discussion of the analysis, we shall briefly review some of the relevant parameters.

1. *Coupling Scheme*

A vector representation of the strong coupling scheme of Bohr and Mottelson is shown in Fig. 1. The nomenclature is discussed in the caption. For the case in which $K = \Omega = \Lambda + \Sigma$ for the intrinsic state (the asymptotic limit approximation), the doubly degenerate states of the two-particle system can be represented as states in which the intrinsic spins are aligned parallel or antiparallel. For these states we adopt

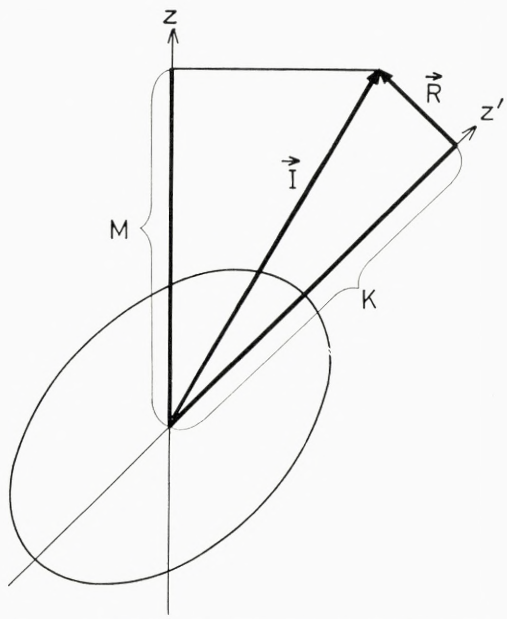

Fig. 1. *Angular momentum coupling scheme for deformed nuclei.* The total angular momentum, I, has the component M along the fixed z-axis and the component K along the nuclear symmetry axis, z'. The collective rotational angular momentum, R, is perpendicular to the nuclear symmetry axis; thus, K is entirely a property of the intrinsic motion.

the shorthand notation $\Sigma = 1$ and $\Sigma = 0$, respectively. In odd-odd nuclei the $\Sigma = 1$ states are lower in energy. At this time no clear-cut exception to this rule is known, and hence it has been used as a guide in assigning odd-odd configurations when the experimental data do not permit a definite assignment.

2. *Rotational Bands*

The energy of a rotational state of spin I in an even-mass nucleus is given by the relationship

$$E_I = E_0 + \frac{\hbar^2}{2\mathfrak{J}} I(I+1),$$

where E_0 is the zero point energy.

The level sequence for levels with $K \neq 0$ is $I, I+1, I+2$, etc. For $K = 0$ three cases occur. $K = 0+$ levels with paired particle configurations have a level sequence $0+$, $2+$, $4+$, etc. Experimentally, $K\pi = 0-$ bands have been found in elements with $A > 220$, which have a spin sequence 1, 3, 5 etc., with no even-spin bands observed as yet(22). Two-quasi-particle levels with $K = 0$ have rotational bands with both odd and even spin states, but the odd and even bands may be displaced depending on the properties of the intrinsic configuration(8, 23, 24, 25).

The moment of inertia $\mathfrak{J}$ appearing in the expression for the energy has been considered elsewhere in detail for the ground-state rotational band(16,17), but moments of inertia of two-quasi-particle states have not as yet been calculated, and we will therefore not attempt a discussion of them.

Associated with the rotational bands are selection and intensity rules which depend on the quantum number K discussed in the caption of Fig. 1. These rules have been given previously(26). In particular, however, transitions of order λ between states in which

$$\lambda < | K_f - K_i |$$

are formally K-forbidden, although they may be allowed by selection rules which depend only on the spin change ΔI and parity change. In addition, as a result of the K quantum number, intensity rules between states of rotational bands occur. For allowed beta decays from an initial state characterized by $I_i \pi_i K_i$ to final states of a rotational band characterized by $I_f \pi_f K_f$, $I_f' \pi_f K_f$, we have the expression

$$\frac{ft\,(I_i \rightarrow I_f)}{ft\,(I_i \rightarrow I_f')} = \frac{\langle I_i\, L\, K_i\, K_f - K_i \mid I_f' K_f \rangle^2}{\langle I_i\, L\, K_i\, K_f - K_i \mid I_f K_f \rangle^2}.$$

Similar K selection and intensity rules are predicted for gamma-ray transitions, and their validity should depend on the extent to which the gamma-ray transitions are not hindered by other intrinsic selection rules and to which the K quantum number is valid.

3. *Selection Rules Associated with the Intrinsic Structure*

Mottelson and Nilsson have discussed the operative selection rules for transitions between states described by the Nilsson wave functions, assuming single-particle transition operators. Alaga has calculated these selection rules when the Nilsson wave functions can be described by the asymptotic limit quantum numbers $(Nn_z\Lambda\Sigma)$(27). These selection rules for beta decay are reproduced in Table III 1. The Nilsson diagrams for the regions $50 \leqslant Z \leqslant 82$ and $82 \leqslant N \leqslant 126$ are reproduced in Fig. 2 and Fig. 3. As has been discussed previously, for two-quasi-particle states connected by single-particle operators, these selection rules apply exactly if the particles in the final and initial states have the same relative coupling and only one

TABLE III 1. *Selection rules for beta transitions in terms of the asymptotic quantum numbers N, n_z, and Λ.*

Transition	ΔK	Operator	$\Delta\Lambda$	Δn_z	ΔN
Allowed (a)	0	1	0	0	0
		σ_z	0	0	0
	1	σ_+	0	0	0
First forbidden (1)	0	z	0	1 −1	1 −1
		$\sigma_z z, \sigma_z \nabla z$	0	1 −1	1 −1
		$\sigma_+(x-iy), \sigma_+ \nabla_-$	−1	0	±1
		$\sigma_-(x+iy), \sigma_- \nabla_+$	1	0	±1
	1	$(x+iy)$	1	0	±1
		$\sigma_+ z, \sigma_+ \nabla z$	0	1 −1	1 −1
		$\sigma_z(x+iy), \sigma_z \nabla_+$	1	0	±1
First forbidden with α-type shape (1*)	0	$\sigma_z z$	0	1 −1	1 −1
		$\sigma_+(x-iy)$	−1	0	±1
		$\sigma_-(x+iy)$	1	0	±1
	1	$\sigma_+ z$	0	1 −1	1 −1
		$\sigma_z(x+iy)$	1	0	±1
	2	$\sigma_+(x+iy)$	1	0	±1

These selection rules were first given by G. ALAGA (ref. 27).

The entries of the table are ordered according to multipolarity and change in angular momentum component ΔK between initial and final states. Column three then contains the corresponding multipole operator. The selection rules in terms of Λ, the component of orbital angular momentum along the nuclear symmetry axis z', N, the total number of nodes in the harmonic oscillator wave function, and n_z, the number of nodal planes perpendicular to the z'-axis are given in columns four, six, and five, respectively.

particle changes in the transition. A second case exists when the relative coupling of the particles is different in the final and initial states. Here, although only one particle changes, and although the matrix elements for the single-particle transition may be non-vanishing, the transition is Λ-forbidden. These transitions are included in groups I and II of Section II. Two-particle transitions are completely forbidden in this scheme. These are considered in group III a of Section II.

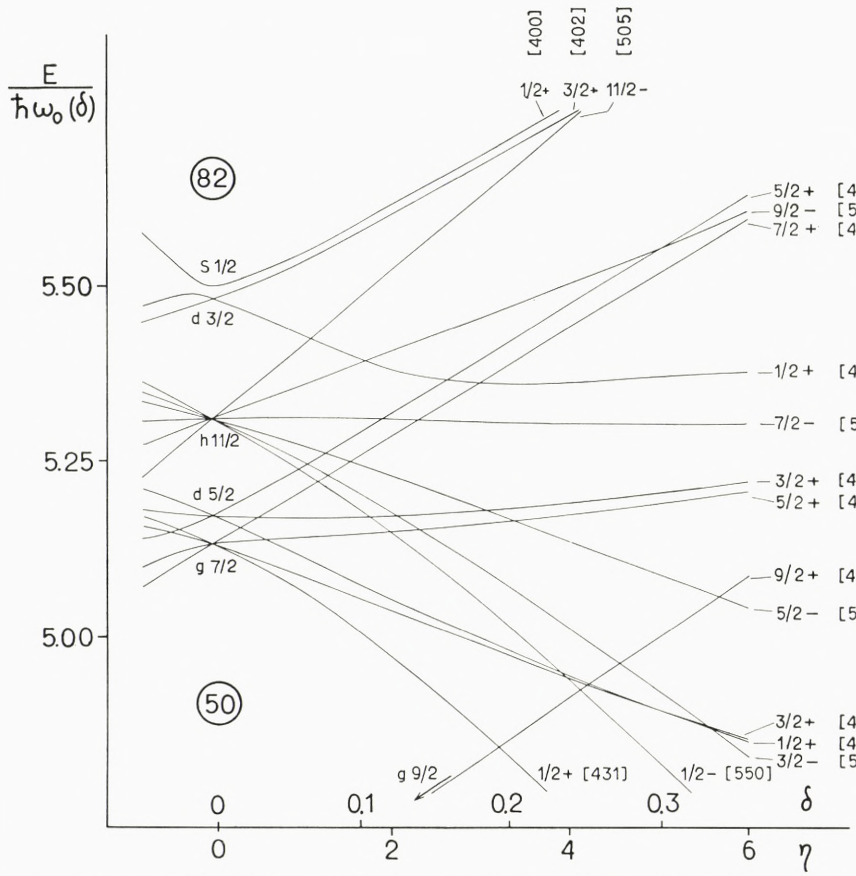

Fig. 2. *Single-particle levels for odd-Z nuclei in the region $50 < Z < 82$.* This figure is a reproduction of fig. 3 in ref. 6, where the parameters which characterize the levels are discussed. Levels used in the present work have been adjusted slightly from the values quoted in ref. 6. (See text and Table II 1).

In discussing the beta transitions we will use the following notations:

a = allowed beta transitions, i. e. $\Delta I = 0$, or 1 (no);
1 = first forbidden beta transitions with $\Delta I = 0$, or 1 (yes);
1* = alpha type first order beta transition, i. e. $\Delta I = 2$ (yes);
u = the transition does not violate the asymptotic selection rules of Table III 1 (i. e. unhindered);
h = the transition violates the selection rules of Table III 1;
Λ = the transition is Λ-forbidden;
K = the transition is K-forbidden. K-forbiddenness is essentially the same as Λ-forbiddenness, but we use K-forbiddenness exclusively for transitions to excited rotational states. (Thus in our notation a transition can be both K- and Λ-forbidden);
F = transitions included in group III of Section II.

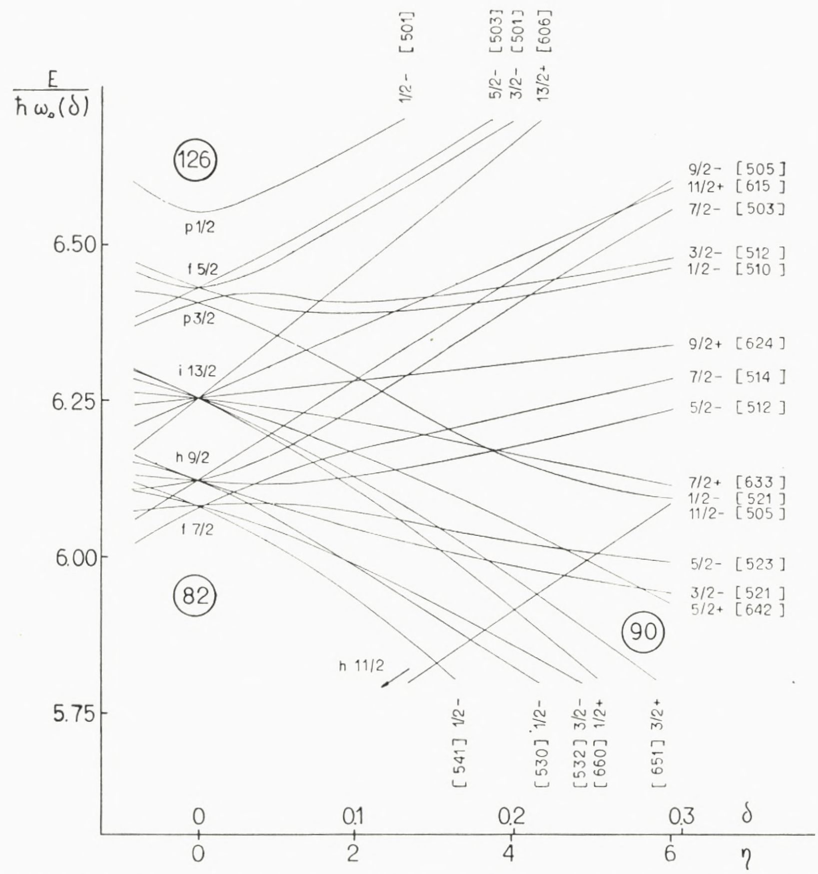

Fig. 3. *Single-particle levels for odd-N nuclei in the region $82 < N < 126$.*. This figure is a reproduction of fig. 4 in ref. 6, where the parameters which characterize the levels are discussed. Levels used in the present work have been adjusted slightly from the values quoted in ref. 6. (See text and Table II 1).

4. *Presentation of Experimental Data*

In the presentation of experimental data we shall discuss separately the level schemes for each mass number. References to the original experimental work are given in the captions to the figures. In general, only experimental references appearing after the 1958 edition of the Table of Isotopes(28) are specifically mentioned. The abbreviations employed are:

half-lives are given in y years, d days, h hours, m minutes, s seconds;
decay energies are indicated where known;
beta decay log ft's are given for each beta group and are indicated by underlining;
excitation energies are given in keV.
The *spin I*, and *parity π* are written at the right side of each level in the order *Iπ*.

To the right of each of the more complex level schemes, the theoretical analysis of the experimental data is given in 3 columns.

The first column gives the assigned K quantum number of the state in addition to the spin I and parity π, in the order $I\pi K$.

In the second column, the type of state is indicated, where the following abbreviations are used:

g = the ground or vacuum state;
c = a state of collective character (see Sec. II. C. 3);
nn = a neutron two-quasi-particle state;
pp = a proton two-quasi-particle state;
i = an intrinsic excitation for which a number of interpretations are possible.

In the third column, the specific configuration for intrinsic states is given in square brackets, viz. ($[Nn_z\Lambda\Sigma \pm N'n'_z\Lambda'\Sigma']$), where the nomenclature is that of the asymptotic limit configurations of the Nilsson wave functions. The intrinsic spin Σ aligned in the direction of the orbital angular momentum directed along the symmetry axis, Λ, is designated $\uparrow$; when it is aligned antiparallel to Λ it is designated $\downarrow$. For odd-odd nuclei the proton state is always listed first; in even-even nuclei the ordering is arbitrary.

In classifying the experimental data shown in the figures we have followed the practice, introduced by Mottelson and Nilsson, of assigning the following somewhat arbitrary grades to the data:

A: Sufficient evidence available to establish the existence of the level and also to indicate quite strongly the spin and parity.
B: Position of the level well-established, but the available data does not uniquely determine the spin and parity.
C: Position of the level based largely on conjecture guided by established systematics or an energy fit with otherwise unassigned gamma rays.

In cases where the complexity of the full level scheme obscures the band structure, we have drawn a separate figure to illustrate the classification of the observed levels into rotational bands.

For simple decay schemes, and for the parent odd-odd nuclei, the format described above has been modified in an obvious way.

5. *Interpretation of Experimental Spectra*

The analysis of data shown in the figures is amplified in Tables III a and b for each mass number. In Table III a, an analysis of the experimental data is made which shows the possible configurations which can be assigned to the level (if more than one exists) and also indicates the classification of the beta transition which populates the state. The calculated energies for the configurations are also listed, as are the observed energies. In cases where the odd-odd configuration assignment is uncertain, multiple assignments for the odd-odd nucleus are listed.

Table III b contains for the specified even-even nucleus the calculated level spectrum. The various configurations are listed in these tables, using the shorthand notation K, $K+1$, etc. In every case, K refers to the last filled orbital, $K-1$, the last filled orbital but one, $K+1$, the first empty orbital, etc. Because the Nilsson levels are in general filled successively, the Nilsson states corresponding to the various indices change regularly. However, as a convenience, the meanings for the index designations for each mass number are indicated at the foot of the corresponding Table III b. The calculated energies are those of the degenerate doublet. The two states of each doublet are listed in the order $\Sigma = 0$, $\Sigma = 1$. In the right-hand columns of Table III b under class are classified all the possible beta transitions from the odd-odd configuration given at the head of the column to the corresponding even-even levels, although in many cases energy considerations preclude the transitions. In several cases, more than one odd-odd configuration is listed, indicating uncertainty in the odd-odd assignments. In the right-hand columns under log *ft* are given the experimental log *ft* values observed. In addition we sometimes include log *ft* values in parentheses. These latter are average values for the same single-particle transition observed in odd-mass nuclei.

B. Level Schemes

A = 152.

The 3− ground state and magnetic moment of 2.0 nm of Eu^{152} support the assignment of the configuration $411\uparrow + 521\uparrow$ to the state. The configuration of the 0− isomeric state is uncertain as three lowlying 0− configurations are possible. The state is most easily assigned as the $\Sigma = 0$ doublet state of the ground-state configuration, but the reported upper limit on the intensity of the allowed $M3$ transition between the isomers casts doubt on this assignment. The best overall fit to the data is achieved if the 0− isomer has the configuration $532\uparrow - 642\uparrow$ (see Table III 2).

The energies of the intrinsic states in Sm^{152} and Gd^{152} have not been calculated (see Section II).

Electron capture decay of 3 − Eu^{152} populates a 1531 keV 2− level with a log ft = 9.0 for the transition. No low lying $K\pi = 2-$ states are expected in the spectrum and the large log ft for this $\Delta I = 1$, no, transition could be explained if the state is a rotational state of a $K = 0-$ or $1-$ band, whence the transition is K-forbidden. This interpretation is further supported by the 1− state at 1511 keV populated by the decay of the 0− isomer. The apparent absence of a 0− state suggests $K\pi = 1-$ as the most probable assignment. The similarity of the decay properties of the 1531 − keV 2− state to the 2− state at 1399 keV in Gd^{154} observed in the decay of Eu^{154} (which has the same ground state as Eu^{152}) suggests that the state is a neutron state. If this is the case, the most probable assignment is $521\uparrow - 642\uparrow$. In Table III 2, the other possible assignment for $K\pi = 1-$ is also listed.

The 3+ level at 1235 keV probably is the first rotational state based on a

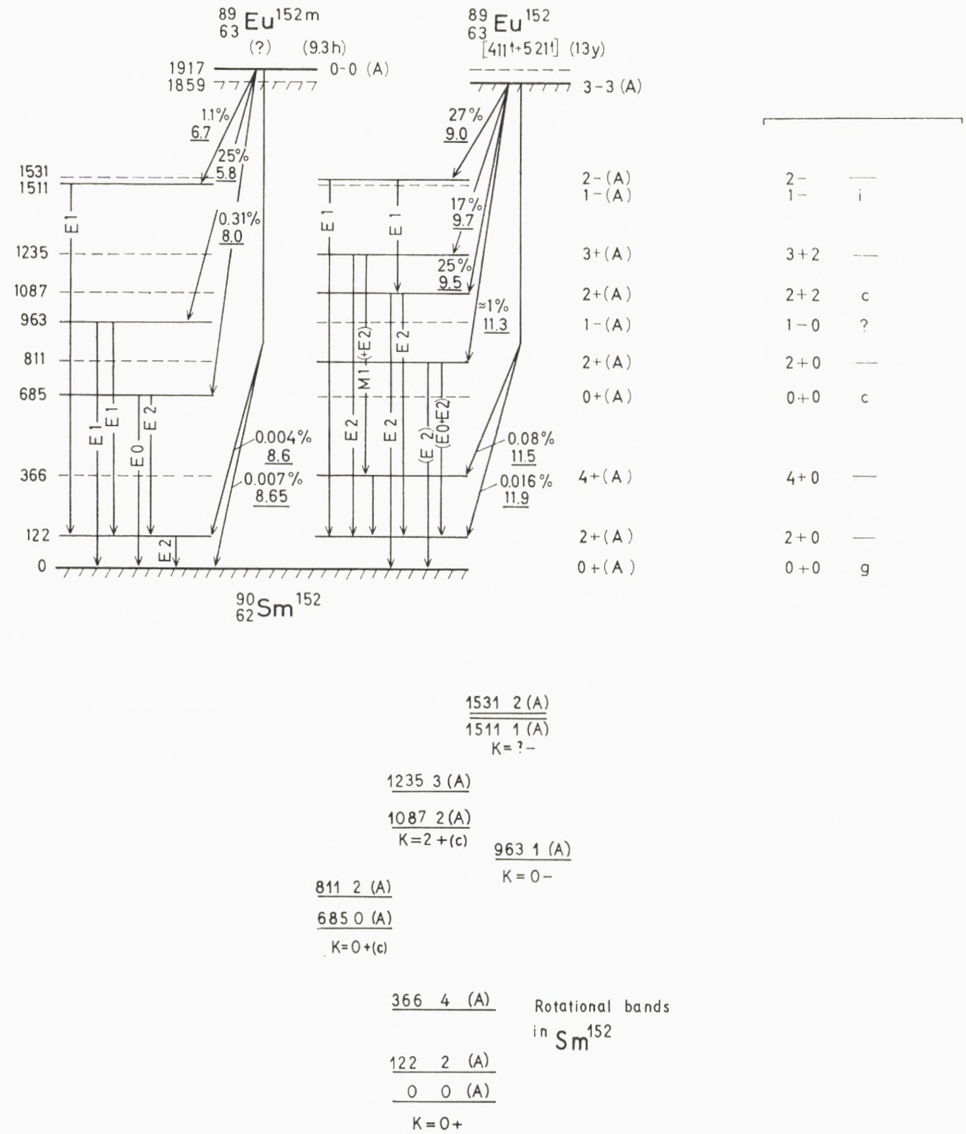

Fig. 4. A = 152.

The spin 0 of the 9.3 h Eu^{152} has been deduced from the upper limit on the magnetic moment of Eu^{152} of $\leqslant 0.004$ nm, reported by V. W. Cohen, J. Schwartz, and R. Novick, Phys. Rev. Letters **2**, 305 (1959). The spin 3 of the 13 *y* Eu^{152} has been deduced from paramagnetic resonance measurements (see SHS). The negative parity assignment to both isomers is as reported by L. Grodzins and A. W. Sunyar, Phys. Rev. Letters **2**, 307 (1959). The energy difference, 50 ± 15 keV, between the Eu isomers has been established by D. Alburger, S. Ofer, and M. Goldhaber, Phys. Rev. **112**, 1998 (1958) who also report a limit on the intensity of the transition between the isomers. A comprehensive review of the experimental data which establish the decay schemes of the 9.3 *h* and 13 *y* Eu^{152} isomers is to be found in the Nuclear Data Sheets (hereinafter called NDS). Additional data on the 9.3 *h* Eu^{152} decay scheme is in I. Marklund, O. Nathan, and O. B. Nielsen, Nuclear Phys. **15**, 199 (1960). Evidence for direct population of the 811 keV 2+ level by 9.3 *h* Eu^{152} has recently been obtained (G. Ewan, priv. comm., 1960; O. Nathan and O. B. Nielsen, priv. comm., 1961).

Eu^{152} (0 – 0) – 1 a, b, c
(3 – 3) – 2

TABLE III 2. ${}^{90}_{62}Sm^{152}$

Experimental		Theoretical		1 a†		1 b†		1 c†		2†	
$I\pi$	E	Final configuration	$I\pi K$	Class	$\log ft$	Class	$\log ft$	Class	$\log ft$	Class	$\log ft$
2 –	1.531	No	2 – 2	. .	. .	. .	. .	. .	. .	. .	9.0
		642 ↑ – 521 ↑ *n*	2 – 1	. .	. .	. .	. .	. .	. .	*aK*(*ah*)	
		651 ↑ – 521 ↑ *n*	2 – 0	. .	. .	. .	. .	. .	. .	*aK*(*ah*)	
		532 ↑ – 411 ↑ *p*	2 – 1	. .	. .	. .	. .	. .	. .	*aK*(*ah*)	
		413 ↓ – 532 ↑ *p*	2 – 0	. .	. .	. .	. .	. .	. .	*aF*	. .
1 –	1.511	642 ↑ – 521 ↑ *n*	1 – 1	*ah*	6.7	*aF*	6.7	*ah*	6.7	. .	. .
		651 ↑ – 521 ↑ *n*	1 – 0	*ah*	. .	*aF*	. .	*aF*	. .	. .	. .
		532 ↑ – 411 ↑ *p*	1 – 1	*ah*	. .	*aF*	. .	*ah*	. .	. .	. .
		413 ↓ – 532 ↑ *p*	1 – 0	*aF*	. .	*ah*	. .	*ah*	. .	. .	. .
2 +	1.087	No	2 + 2	. .	. .	. .	. .	. .	. .	. .	9.5
		(collective)	2 + 2	. .	. .	. .	. .	. .	. .	1 ?	. .
1 –	.963	642 ↑ – 523 ↓ *n*	1 – 0	*aF*	5.8	*ah*	5.8	*ah*	5.8	. .	. .
		651 ↑ – 521 ↑ *n*	1 – 0	*ah*	. .	*aF*	. .	*aF*	. .	. .	. .
		413 ↓ – 532 ↑ *p*	1 – 0	*aF*	. .	*ah*	. .	*ah*	. .	. .	. .
		(collective)	1 – 0	. .	. .	. .	. .	. .	. .	. .	. .
2 +	.811	(collective)	2 + 0	. .	. .	. .	. .	. .	. .	1 ?	11.3
4 +	.366	ground	4 + 0	. .	. .	. .	. .	. .	. .	1*K*(1*u*)	11.5
2 +	.122	ground	2 + 0	1**h*(1*u*)	8.6	1**h*(1*u*)	8.6	1**h*(1*u*)	8.6	1*K*(1*u*)	11.9
0 +	0	ground	0 + 0	–	8.7	–	8.7	–	8.7	. .	. .

† 1 a) 411 ↑ – 521 ↑ 1 b) 413 ↓ – 523 ↓ 1 c) 532 ↑ – 642 ↑ 2) 411 ↑ + 521 ↑

$K = 2+$ state at 1087 keV. No low lying $2+$ states are expected in the intrinsic spectrum, suggesting the state is collective. The assignment of the $2+$ state at 811 keV as a rotational state based on a $0+$ level at 685 keV is established by the mixed $E0 + E2$ transition to the $2+$ state of the ground rotational band. The $0+$ state is probably best described as a collective state.

The 963 keV $1-$ state is assigned as a $K\pi = 0-$ state on the basis of its gamma-ray branching to the ground-state band. Two intrinsic excitations with $K\pi = 0-$ are expected, but a rough estimate of the excitation energies of these levels gives a considerably higher energy than that observed.

A = 154.

The magnetic moment of 2.1 nm and spin 3 of the Eu^{154} ground state are consistent with the assignment of the configuration 411 ↑ + 521 ↑ (21). The similarity of both magnetic moment and decay scheme of Eu^{124} to that of $3-$ Eu^{152} is additional evidence for the assignment of the same ground state to both.

In Gd^{154}, with 90 neutrons, the same problem exists in calculating the neutron spectrum as in Sm^{152} (see Section II).

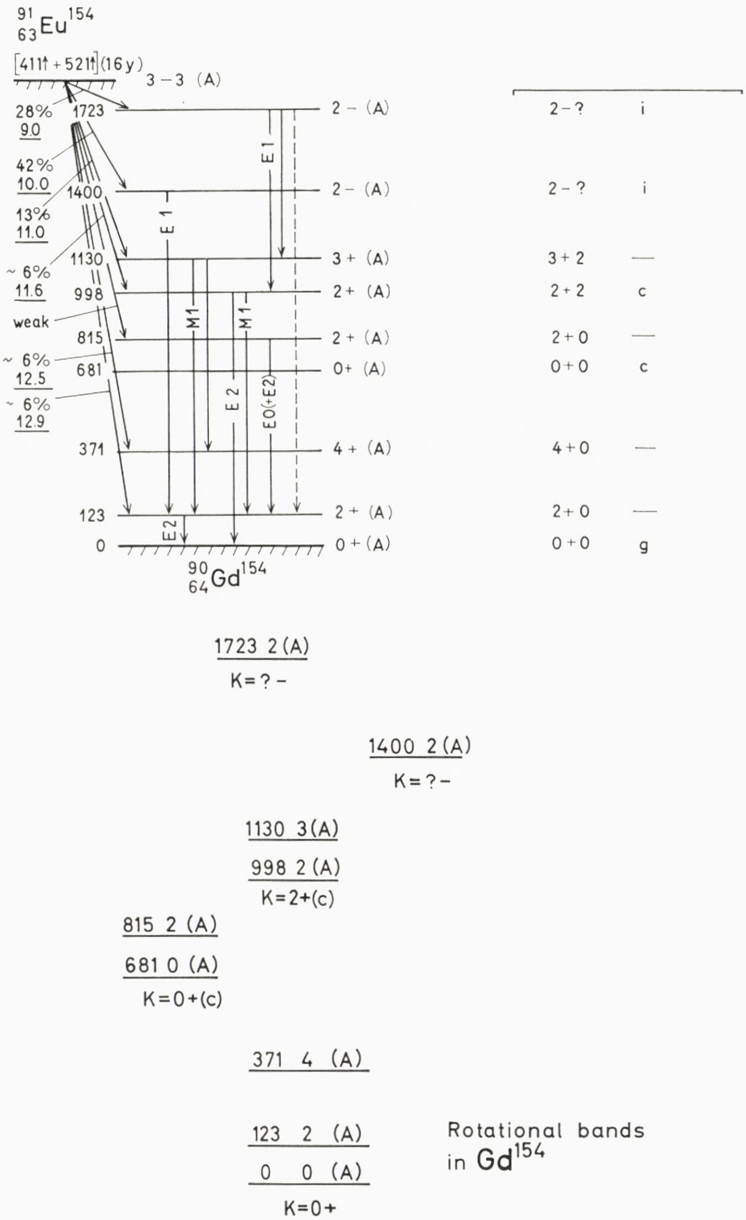

Fig. 5. A = 154.

The spin 3 and magnetic moment of 2.1 nm of Eu^{154} have been deduced from paramagnetic resonance measurements (see SHS). The basic decay scheme of 16 *y* Eu^{154} was established by J. O. Juliano and F. S. Stephens, Phys. Rev. **108**, 341 (1957); more detailed references to other experimental work are to be found in the NDS. The 815 keV 2+ level has been observed in Eu^{154} decay (unpublished results quoted in O. Nathan and S. Hultberg, Phys. Rev. **10**, 118 (1959)), but the assignment of the 681 keV 0+ and 815 keV levels is based on preliminary results on the decay scheme of Tb^{154} (B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **123**, 1758 (1961) hereinafter referred to as HHM 61; M. Jørgensen, O. Nathan, and O. B. Nielsen, priv. comm. (1960). The 2− assignment of the 1400 and 1723 keV levels has been established by P. Derbrunner and W. Kündig, Helv. Phys. Acta **33**, 395 (1960), and R. Stiening and M. Deutsch, Phys. Rev. **121**, 1484 (1961).

Eu^{154} (3 − 3) −1 TABLE III 3. $^{90}_{64}Gd^{154}$

Experimental		Theoretical			1†	
I π	E	Final configuration	IπK	E	Class	log ft
2 −	1.723	411 ↑ − 523 ↑ p	2 − 2	2.0	$a\Lambda(2)$	9.0
		651 ↑ − 521 ↑ n	2 − 0	. .	$aK(ah)$	. .
		642 ↑ − 521 ↑ n	2 − 1	. .	$aK(ah)$	. .
		532 ↑ − 411 ↑ p	2 − 1	1.7	$aK(ah)$	. .
		532 ↑ − 413 ↓ p	2 − 0	1.9	aF	. .
		413 ↓ − 523 ↑ p	2 − 1	1.9	aF	. .
2 −	1.400	same as 1.723	. .	. .	. .	10.0
2 +	.998	413 ↓ − 411 ↓ p	2 + 2	2.4	$1\Lambda(1h)$	11.6
		(collective)	2 + 2	. .	1 ?	. .
2 +	.815	(collective)	2 + 0	. .	1 ?	large
4 +	.371	ground	4 + 0	. .	$1K(1u)$	12.5
2 +	.123	ground	2 + 0	. .	$1K(1u)$	12.9

† 1) 411 ↑ + 521 ↑

The interpretation of the levels in Gd^{154} at 1400, 1130, 998, 815, and 681 keV seems identical to that of the 1531, 1235, 1087, 811, and 685 keV levels in Sm^{152}. The decay of the 2 − level at 1723 keV to the $K\pi = 2+$ states at 998 and 1130 keV suggests $K\pi = 1-$ or $2-$ for the state. The only $K\pi = 2-$ state is expected at somewhat higher energy. The log $ft = 9.0$ for the beta transition to this state indicates a retarded transition, but this does not help appreciably in classifying the level, because it is predicted that transitions to all low lying negative parity states are retarded.

A = 156.

The assignment of the 3− configuration 411 ↑ + 521 ↑ to Tb^{156} seems reasonable on the basis of the decay scheme. The 4− level at 2042 keV seems better described as the intrinsic state 521 ↑ + 642 ↑ (n) than the state 532 ↑ + 411 ↑ (p) because of the somewhat lower energy predicted for the neutron state, although the log ft's for the transitions 411 ↑ (p)→642 ↑ (n) and 532 ↑ (p)→642 ↑ (n) are of the same order of magnitude. Only one intrinsic 3 − state is available in the low energy spectrum, 651 ↑ + 521 ↑ (n), and the observed log ft for the single-particle transition 411 ↑ (p)→ 651 ↑ (n) is 7.0, consistent with the log ft for the Tb^{156}→Gd^{156} transition. However, the 3 − state appears at somewhat lower energy than that calculated for the configuration listed. No 5 + intrinsic states are expected, so the 1620 keV state seems best assigned as a rotational state. The 4 + state at 1507 keV is quite naturally assigned as the $\Sigma = 0$ member of the K, $K+1$ proton state 413 ↓ + 411 ↑. The 1154 and 1246 keV states are well described as a $K\pi = 2+$ rotational band, probably collective in nature, as no low lying 2 + states are expected in the spectrum.

The Eu^{156} ground state seems fairly well described by the 1 − configuration 413 ↓ − 521 ↑. The transition to the Gd^{156} ground state is somewhat more strongly

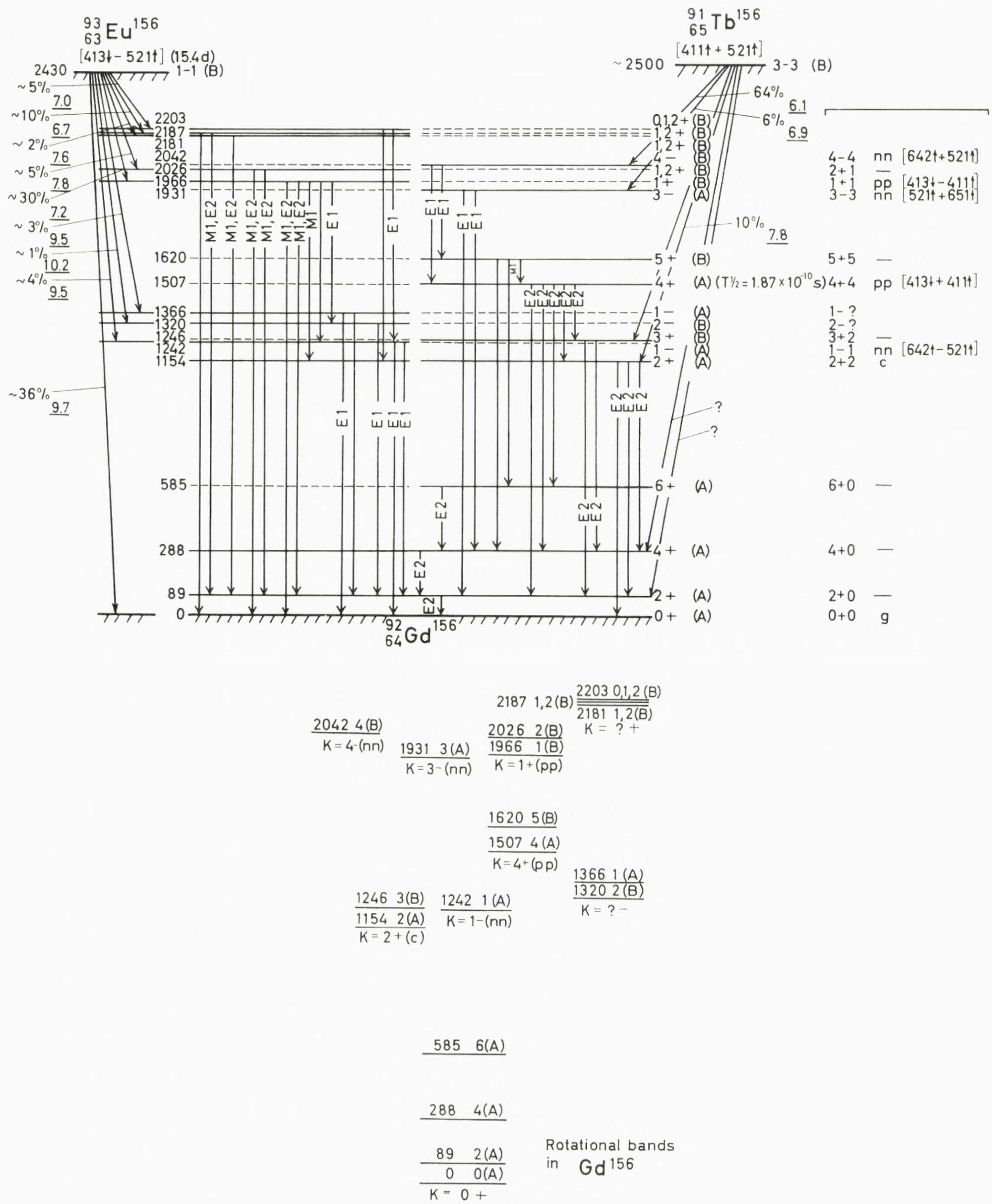

Fig. 6. A = 156.

The decay scheme of Eu^{156} has been proposed by G. Ewan, J. S. Geiger, and R. L. Graham (to be published); many features of this decay scheme are supported by the gamma-ray measurements of J. E. Cline and R. L. Heath, Nuclear Phys. **22**, 598 (1961). The Tb^{156} decay scheme is as reported by P. G. Hansen, O. B. Nielsen, and R. K. Sheline, Nuclear Phys. **12**, 389 (1959); S. Ofer, Phys. Rev. **115**, 412 (1959). The lifetime of the 1507 keV level has been reported by R. E. Bell and M. H. Jørgensen, Nuclear Phys. **12**, 413 (1959). References to earlier measurements are quoted in the references above, and in SHS and the NDS.

Eu^{156} (1 − 1) − 1

Tb^{156} (3 − 3) − 2

Table III 4 a. $^{92}_{64}Gd^{156}$

Experimental		Theoretical			1†		2†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft	Class	log ft
0,1,2+	2.203	–	..	..	1?	7.0	..	..
1,2+	2.187	–	..	..	1?	6.7	..	..
1,2+	2.181	–	..	..	1?	7.6	..	..
4−	2.042	642 ↑ + 521 ↑ n	4 − 4	1.5	..	..	ah	6.1
		532 ↑ + 411 ↑ p	4 − 4	1.7	..	..	ah	..
1,2+	2.026	rot. state 1.966	2 + 1	..	..	7.8	..	..
1+	1.966	413 ↓ − 411 ↑ p	1 + 1	1.4	1u	7.2	..	..
		523 ↓ − 521 ↑ n	1 + 1	1.7	1u	..	..	..
3−	1.931	521 ↑ + 651 ↑ n	3 − 3	2.0	..	..	ah	6.9
4+	1.507	413 ↓ + 411 ↑ p	4 + 4	1.4	..	..	1h	8.0
		523 ↓ + 521 ↑ n	4 + 4	1.7	..	..	1h	..
1−	1.366	521 ↑ − 651 ↑ n	1 − 0	..	ah	9.5	..	..
		642 ↑ − 521 ↑ n	1 − 1	1.5	ah	..	..	..
		532 ↑ − 411 ↑ p	1 − 1	1.7	aF	..	..	..
2−	1.320	521 ↑ − 651 ↑ n	2 − 0	..	..	10.2	..	..
		642 ↑ − 521 ↑ n	2 − 1	..	..	..	..	..
		532 ↑ − 411 ↑ p	2 − 1	..	..	..	..	..
1−	1.242	same as 1.366	..	..	..	9.5	..	..
2+	1.154	(collective)	2 + 2	..	1?	–	1?	7.8
4+	.288	ground	4 + 0	..	..	..	1K(1u)	large
2+	.089	ground	2 + 0	..	1h	large	1K(1u)	large
0+	0	ground	0 + 0	..	1h	9.7	..	..

† 1) 413 ↓ − 521 ↑ 2) 411 ↑ + 521 ↑

retarded (log ft = 9.5) than the single-particle rate observed in Eu^{155} decay, where a log ft = 8.7 is observed for the transition. It should be noted that the 0+ assignment proposed for Eu^{156} by G. Ewan et al. (ref. 29) can also explain many features of the decay scheme if the 0+ configuration 413 ↓ − 642 ↑ is assumed. A choice between these two assignments does not seem possible on the basis of the present data.

The levels populated by Eu^{156} cannot be interpreted unambiguously. Low lying $\Sigma = 0$ negative parity states are expected on the basis of the $\Sigma = 1$ 3− and 4− states observed in Tb^{156} decay, but the data can be interpreted in a number of ways. The observed log ft's for transitions to the 1966 keV and 2187 keV levels suggest that these levels have the configurations 411 ↑ − 413 ↓ (p) and 521 ↑ − 523 ↓ (n), although which is which is not clear. The experimental assignment of the other levels is at present uncertain.

A = 160.

The $K\pi$ = 3− assignment and magnetic moment of 1.6 ± 0.25 of Tb^{160} are in agreement with the assignment of the configuration 411 ↑ + 521 ↑ to Tb^{160} (ref. 30).

$^{92}_{64}$Gd^{156}

TABLE III 4 b.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{91}_{65}$Tb^{156} 3 − $p = K+1$; $n = K$ Class	log ft	$^{93}_{63}$Eu^{156} 1 − $p = K$; $n = K$ Class	log ft
			neutron levels				
$K, K+1$	1 −	1.5	..	..	..	ah	9.5 (7.5)
	4 −	..	2.042	ah	6.1	..	..
K, K .... / $K+1, K+1$	0 +	1.6	..	..	..	1h	..
$K, K+2$	4 +	1.7	1.507	1h	large	..	..
	1 +		1.966	1*Λ(1*h)	..	1u	7.2
$K+1, K+2$	5 −	1.9	..	..	..	..	..
	0 −	..	..	..	..	aF	..
$K, K+3$	4 −	2.0	..	aΛ(3)	..	..	..
	7 −		..	..	..	..	..
$K-1, K+1$	1 +	2.0	..	1*F	..	1F	..
	4 +	..	..	1F	..	..	..
$K-1, K$	0 −	2.0	..	..	..	ah	..
	3 −	..	1.931	ah	6.9	..	..
$K+1, K+3$	3 −	2.1	..	aF	..	..	..
	8 −	..	..	..	..	..	..
$K-1, K+2$	4 −	2.1	..	aF	..	..	..
	1 −	..	..	..	..	aF	..
$K-2, K$	1 −	2.3	..	..	..	ah	..
	2 −	..	..	ah	..	aΛ(ah)	..

$K-2 = 660\uparrow$ $K-1 = 651\uparrow$ $K = 521\uparrow$ $K+1 = 642\uparrow$ $K+2 = 523\downarrow$ $K+3 = 505\uparrow$

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{91}_{65}$Tb^{156} Class	log ft	$^{93}_{63}$Eu^{156} Class	log ft
			proton levels				
$K, K+1$	4 +	1.4	1.507	1h	large (8.5)	..	..
	1 +		1.966	1*Λ(1*h)	..	1u	7.2
K, K .... / $K+1, K+1$	0 +	1.6	..	..	..	1h	..
$K-1, K+1$	1 −	1.7	..	..	..	aF	..
	4 −	..	2.042	ah	6.1	..	..
$K-1, K$	5 −	1.9	..	..	..	..	..
	0 −	..	..	..	..	ah	..
$K, K+2$	6 −	1.9	..	..	..	..	..
	1 −	..	..	..	..	a(2)	..
$K+1, K+2$	2 −	2.0	..	aΛ(2)	..	aF	..
	5 −	..	..	..	..	..	..
$K-1, K+2$	1 +	2.0	..	1*F	..	1F	..
	6 +	..	..	..	..	..	..
$K, K+3$	2 +	2.4	..	1F	..	1u	(6.2)
	3 +		..	..	..	1*Λ(1u)	..
$K+1, K+3$	2 +	2.5	..	1u	..	1F	..
	1 +	..	..	1*Λ(1u)	..	1F	..

$K-1 = 532\uparrow$ $K = 413\downarrow$ $K+1 = 411\uparrow$ $K+2 = 523\uparrow$ $K+3 = 411\downarrow$

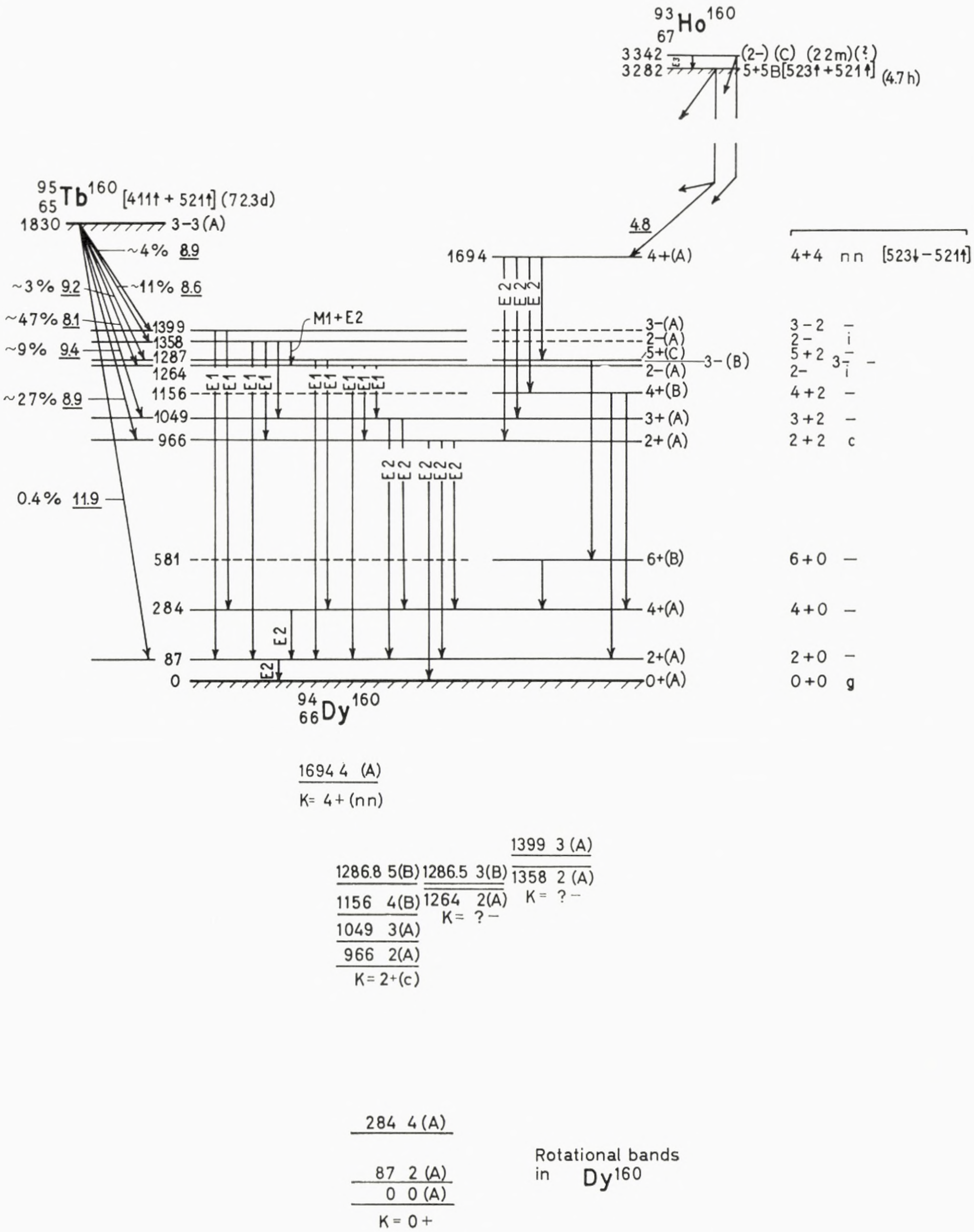

Fig. 7. A = 160.

The decay scheme of Tb^{160} has been extensively studied. The newest experimental data and reviews of previous work are reported by C. E. Johnson, J. F. Schooley, and D. A. Shirley, Phys. Rev. **120**, 2108 (1960); G. T. Ewan, R. L. Graham, and J. S. Geiger, Nuclear Phys. **22**, 610 (1961). (See also N. A. Voinova, B. S. Dzhelepov, N. N. Zhukovskii, and Yu. V. Khol'nov, Izvest. Akad. Nauk SSSR, Ser Fiz. **24**, 852 (1960); A. V. Kogan, V. D. Kul'kov, L. P. Nikitin, N. M. Reinov, I. A. Sokolov, and M. F. Stelmakh, Programma i Tezisy Dokladov 11 Ežegdnogo Soveščanija po Jadernoj Spektroskopii v Rige, Akad. Nauk SSSR, p. 89).

Studies of the Ho^{160} decay scheme have been made. (See the NDS; also K. S. Toth and J. O. Rasmussen, Phys. Rev. **115**, 150 (1959); B. S. Dzhelepov, I. Zvol'skii, M. K. Nikitin, and V. A. Sergienko, Izvest. Akad. Nauk SSSR, **25**, 1246 (1961); E. P. Grigoriev, B. S. Dzhelepov, V. Zvol'ska, and A. V. Zolotavin, Programma i Tezisy Dokladov 11 Ežegdnogo Soveščanija po Jadernoj Spektroskopii v Rige, Akad. Nauk SSSR, p. 55. These results all indicate the very great complexity of the decay scheme. However, the 4+ level at 1694 keV is populated by an allowed, unhindered $\beta+$ group (E. P. Grigoriev, B. S. Dzhelepov, and A. V. Zolotavin, Izvest. Akad. Nauk, SSSR, Ser. Fiz. **22**, 821 (1958), for which reason we include it in the Dy^{160} level scheme.

Note that the 1694 keV level configuration should be 523↓ + 521↑.

Tb^{160} $(3-3)-1$
Ho^{160} $(5+5)-2$

TABLE III 5a. $^{94}_{66}Dy^{160}$

Experimental		Theoretical			1†		2†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft	Class	log ft
4+	1.694	$523\downarrow + 521\uparrow n$	4+4	1.6	..	..	au	≈ 4.8
3−	1.399	rot. state 1.358	3−?	..	–	8.9	..	..
2−	1.358	$521\uparrow - 642\uparrow n$	2−1	1.5	$aK(ah)$	8.6	..	..
		$411\uparrow - 523\uparrow p$	2−2	1.4	$a\Lambda(ah)$	..	..	..
3−	1.287	rot. state 1.264	..	..	–	9.2	..	..
2−	1.264	same as 1.358	..	..	..	8.1	..	..
3+	1.049	rot. state .966	3+2	–	–	9.4	..	..
2+	.966	$411\uparrow + 411\downarrow p = 411\downarrow p$	2+2	2.0	$1u$	8.9	..	..
		(collective)	2+2	..	1?	..	..	..
2+	.087	ground	2+0	–	$1K(1u)$	11.9	..	..

† 1) $411\uparrow + 521\uparrow$ 2) $523\uparrow + 521\uparrow$

Two rotational bands of negative parity are observed in Dy^{160} although the K quantum numbers of the bands are not clear. Possible assignments of the bands are indicated in Table III 5 a. It is interesting to note that all transitions are strongly forbidden, as observed, and that states to which faster transitions could occur all appear to have somewhat higher energy than the decay energy of Tb^{160}. The 966 and 1049 keV levels are well established as members of a $K\pi = 2+$ band, and their low energy relative to the calculated 2+ state energies supports their assignment as collective states.

The similarity of the (3−) Eu^{152}, Eu^{154}, and Tb^{160} decay schemes should be noted. In all these isotopes the ground-state configuration is identical, all have relatively little decay energy, and all decay by highly retarded (probably K-forbidden) transitions to apparently the same levels in the daughter nuclei. It would be interesting, if this analogy were pursued further experimentally, to see whether the decays of Eu^{152} and Eu^{154} also populate 3− levels above the 1531 keV and 1400 and 1723 keV levels, respectively.

Much additional information on the levels of Dy^{160} will be provided by further studies of Ho^{160} decay, as preliminary measurements have indicated an extremely complex spectrum. We have included a level at 1694 keV in the illustrated decay scheme, which, on the basis of the $E2$ transitions from it to the 2+, 3+, and 4+ levels at 966, 1049, and 1156 keV is 4+. The log $ft = 4.8$ from the 4.7 h Ho^{160} to this level is clearly associated with the $523\uparrow(p)\rightarrow 523\downarrow(n)$ transition, establishing the 4+ level as the $523\downarrow + 521\uparrow$ neutron configuration and the Ho^{160} isomer as the 5+ configuration $523\uparrow + 521\uparrow$.

A = 162.

The log $ft = 4.6$ observed in the decays of both members of the Ho^{162} isomeric pair clearly establishes that the decays involve the transition $523\uparrow(p)\rightarrow 523\downarrow(n)$.

$^{94}_{66}\mathrm{Dy}^{160}$

Table III 5 b.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{95}_{65}\mathrm{Tb}^{160}$ $p=K$ Class	$3-$ $n=K$ log ft	$^{93}_{67}\mathrm{Ho}^{160}$ $p=K+1$ Class	$5+$ $n=K$ log ft
			neutron levels				
$K, K+1$	$1-$	1.5	..	..	..	..	..
	$4-$	..	..	ah	..	$1h$	..
$K, K+2$	$4+$	1.6	1.694	$1h$	..	au	4.8 (5.0)
	$1+$	..	..	$1^*A(1^*h)$	..	..	..
K, K / $K+1, K+1$	$0+$	1.6	..	..	..	..	..
$K-1, K+1$	$1+$	1.7	..	1^*F	..	..	..
	$4+$	..	..	$1F$	..	aF	..
$K+1, K+2$	$5-$	1.7	..	..	..	$1F$	..
	$0-$	..	..	..	..	..	..
$K-1, K+2$	$4-$	1.8	..	aF	..	$1F$	..
	$1-$	..	..	..	..	..	..
$K-1, K$	$0-$	1.8	..	..	..	..	..
	$3-$	..	..	ah	..	1^*h	..
$K, K+3$	$2-$	2.6	..	$aA(ah)$	..	..	..
	$5-$	..	..	..	..	$1u$	(6.1)
$K+1, K+3$	$1+$	2.7	..	1^*F	..	..	..
	$6+$	..	..	..	..	aF	..

$K-1 = 651\uparrow$ $K = 521\uparrow$ $K+1 = 642\uparrow$ $K+2 = 523\downarrow$ $K+3 = 633\uparrow$

State	$K\pi$	Calc.	Exper.	$^{95}_{65}\mathrm{Tb}^{160}$ Class	log ft	$^{93}_{67}\mathrm{Ho}^{160}$ Class	log ft
			proton levels				
$K, K+1$	$2-$	1.4	..	$aA(2)$	..	..	..
	$5-$	..	..	..	..	$1u$	..
$K-1, K+1$	$6-$	1.7	..	..	..	$1h$	(8)
	$1-$	..	..	..	..	..	..
K, K / $K+1, K+1$	$0+$	1.8	..	..	..	..	..
$K, K+2$	$2+$	2.0	..	$1u$	(6.3)	..	..
	$1+$	..	..	$1^*A(1u)$	..	..	..
$K-2, K+1$	$1+$	2.0	..	1^*F	..	..	..
	$6+$	..	..	..	..	ah	..
$K-1, K$	$4+$	2.1	..	$1h$	..	aF	..
	$1+$	..	..	$1^*A(1h)$	..	..	..
$K-1, K+2$	$2+$	2.2	..	$1F$	..	..	..
	$3+$	..	..	$1F$	..	..	..
$K+1, K+2$	$4-$	2.2	..	aF	..	$1u$	(7.0)
	$3-$	..	..	aF	..	$1^*A(1u)$	..
$K+1, K+3$	$7-$	2.9	..	..	..	1^*h	..
	$0-$	..	..		..	..	..

$K-2 = 532\uparrow$ $K-1 = 413\downarrow$ $K = 411\uparrow$ $K+1 = 523\uparrow$ $K+2 = 411\downarrow$ $K+3 = 404\downarrow$

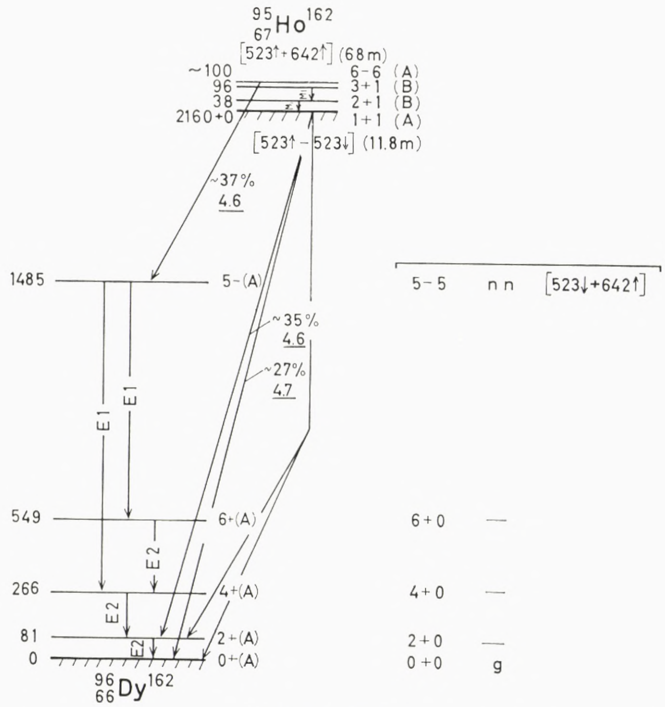

Fig. 8. A = 162.
The isomers of Ho^{162} and the decay schemes have recently been established by M. Jørgensen, O. B. Nielsen, and O. Skilbreid, Nuclear Phys. **24**, 443 (1961). The configuration assignments were originally proposed by these authors.

Ho^{162} (6 − 6) − 1
(1 + 1) − 2 Table III. 6 a. $^{96}_{66}Dy^{162}$

Experimental		Theoretical			1†		2†	
Iπ	*E*	Final configuration	IπK	*E*	Class	log *ft*	Class	log *ft*
5 −	1.485	523 ↓ + 642 ↑	5 − 5	1.3	*au*	4.6	..	..
2 +	.081	ground	2 + 0	—	..	..	*au*	≈ 4.6
0 +	0	ground	0 + 0	—	..	..	*au*	≈ 4.7

† 1) 523 ↑ + 642 ↑ 2) 523 ↑ − 523 ↓

A 6 − assignment for the upper isomer is definite from its allowed decay to the 5 − level in Dy^{162} at 1485 keV, which in turn is clearly established from its decay properties. The 6 − Ho^{162} configuration can therefore only be 523 ↓ + 642 ↑, and the 5 − state the neutron state 523 ↓ + 642 ↑. The allowed decay to the Dy^{162} ground state clearly establishes the 11.8 min isomer as the 1 + state 523 ↑ − 523 ↓. These assignments have previously been made by M. Jørgensen, O. B. Nielsen, and O. Skilbreid(31).

$^{96}_{66}Dy^{162}$

TABLE III 6 b.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{95}_{67}Ho^{162}$ $p = K+1$; $1+$ $n = K+1$ Class	$^{95}_{67}Ho^{162}$ $p = K+1$; $1+$ $n = K+1$ log ft	$^{95}_{67}Ho^{162}$ $p = K+1$; $6-$ $n = K$ Class	$^{95}_{67}Ho^{162}$ $p = K+1$; $6-$ $n = K$ log ft
neutron levels							
$K, K+1$	5 −	1.3	1.485	..	..	au	4.6 (5.2)
	0 −	..	..	$1u$	..	..	..
K, K ; $K+1, K+1$	0 +	1.4	..	au	..	..	..
$K-1, K+1$	4 +	1.6	..	..	..	1^*F	..
	1 +	..	..	$a(2)$	..	..	..
$K-1, K$	1 −	1.8	..	$1F$	..	..	..
	4 −	..	..	..	..	..	..
$K-2, K+1$	4 −	1.9	..	..	..	..	..
	1 −	..	..	1^*h	..	..	..
$K-2, K$	1 +	2.1	..	aF	..	..	..
	4 +	..	..	..	..	1^*h	..
$K, K+2$	1 +	2.2	..	aF	..	..	..
	6 +	..	..	..	..	$1u$	..
$K+1, K+2$	6 −	2.2	..	..	..	aF	..
	1 −	..	..	$1u$	..	..	..
$K, K+3$	3 −	2.3	..	1^*F	..	..	..
	2 −	..	..	$1F$	..	..	..

$K-2 = 651\uparrow$, $K-1 = 521\uparrow$, $K = 642\uparrow$, $K+1 = 523\downarrow$, $K+2 = 633\uparrow$, $K+3 = 521\downarrow$

State	$K\pi$	Calc.	Exper.	Class	log ft	Class	log ft
proton levels							
$K, K+1$	2 −	1.4	..	$1h$	..	..	..
	5 −	..	..	..	..	ah	..
$K-1, K+1$	6 −	1.7	..	..	..	ah	..
	1 −	..	..	$1u$	..	..	..
K, K ; $K+1, K+1$	0 +	1.8	..	au	..	..	..
$K, K+2$	2 +	2.0	..	aF	..	..	..
	1 +	..	..	aF	..	..	..
$K-2, K+1$	1 +	2.0	..	ah	..	..	..
	6 +	..	..	..	..	$1u$	..
$K-1, K$	4 +	2.1	..	..	..	..	..
	1 +	..	..	aF	..	..	..
$K-1, K+2$	2 +	2.2	..	aF	..	..	..
	3 +	..	..	..	..	..	..
$K+1, K+2$	4 −	2.2	..	..	..	..	..
	3 −	..	..	1^*h	..	..	..
$K+1, K+3$	7 −	2.9	..	..	..	ah	..
	0 −	..	..	$1u$	..	..	..

$K-2 = 532\uparrow$ $K-1 = 413\downarrow$ $K = 411\uparrow$, $K+1 = 523\uparrow$, $K+2 = 411\downarrow$, $K+3 = 404\downarrow$

A = 164.

The Ho^{164} isomers are identical to the Ho^{162} isomers, except the lower decay energy rules out the beta decay from the 6 − isomer to the 5 − state in Dy^{164}. The assignments have previously been proposed by M. JØRGENSEN, O. B. NIELSEN, and O. SKILBREID(32). The allowed, unhindered decay of Tm^{164} to Er^{164} seems clearly to establish the 2 m Tm^{164} isomer as having the 1 + configuration 523 ↑ − 523 ↓.

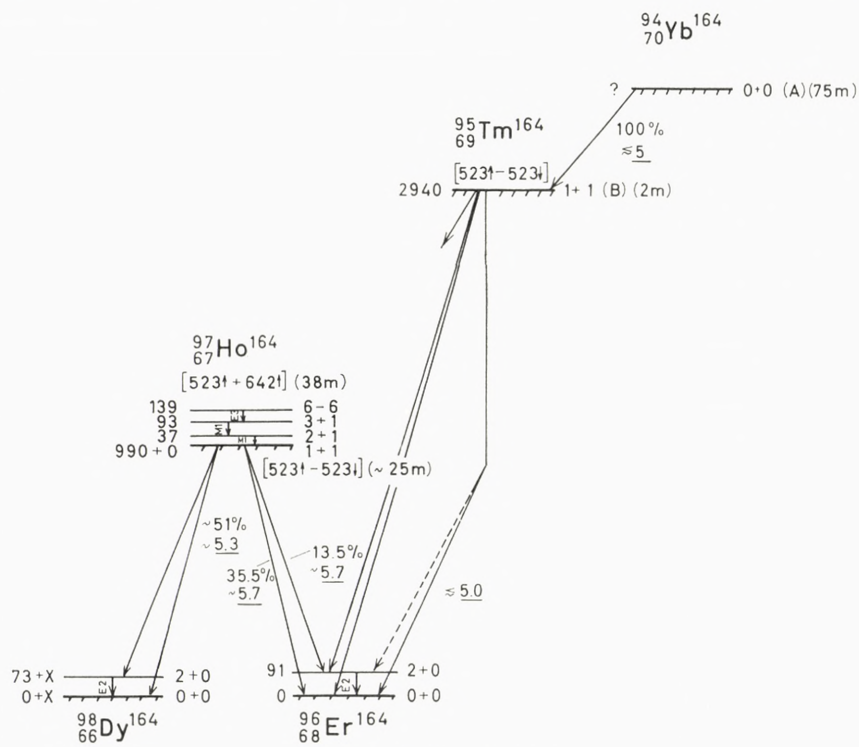

Fig. 9. A = 164.

The presence of two isomers of Ho^{164} has been established by M. JØRGENSEN, O. B. NIELSEN, and O. SKILBREID (priv. comm., Feb. 1961), who assigned the decay scheme of Ho^{164} as reported by H. N. BROWN and R. A. BECKER, Phys. Rev. **96**, 1372 (1954) to arise from the decay of the 1+ isomer. The configuration assignments were originally proposed by M. JØRGENSEN et al.

The data on the $Yb^{164} \rightarrow Tm^{164} \rightarrow Er^{164}$ decay chain are as reported by A. ABDURAZAKOV, K. GROMOV, B. DALKHSUREN, B. DZHELEPOV, I. LEVENBERG, A. MURIN, YU. NORSEYEV, V. POKROVSKY, V. CHUMIN, and I. YUTLANDOV, Nuclear Phys. **21**, 164 (1960).

A = 166.

The levels in Ho^{166} populated by Dy^{166} decay seem well described by rotational states based on a $K = 0-$ state. The band manifests a displacement of the odd-spin levels with respect to those with even spin. An intrinsic 1 + level at 428 keV is populated with a log ft = 4.8, clearly identifying the level as the configuration 523 ↑ − 523 ↓. Of the levels reported from Ho^{165} (n, γ), Ho^{166} reactions, the 3 + level at 194 keV

Ho^{164} (1+1)−1
Tm^{164} (1+1)−2

TABLE III 7.

$^{98}_{66}Dy^{164}$ $^{96}_{68}Er^{164}$

Experimental		Theoretical			1†		2†	
Iπ	E	Final configuration	IπK	E	Class	log ft	Class	log ft
2+	.091	ground (Er^{164})	2+0	–	au	≈5.4	..	..
0+	0	ground (Er^{164})	0+0	–	au	≈5.7	au	$\lesssim 5.0$
2+	.073	ground (Dy^{164})	2+0	–	au	≈5.3	..	..
0+	0	ground (Dy^{164})	0+0	–	au		..	..

† 1) 523↑−523↓ 2) 523↑−523↓

is assigned the configuration 523↑−521↓, and the 4− level seems well described as a rotational state of the 0− band. The $K\pi = 7-$, $\Sigma = 1$, member of the Ho^{166} ground state doublet (the long lived Ho^{166}) is essentially degenerate with the $\Sigma = 0$ state, the 7− state having an energy 9 ± 33 keV lower than the 0− state.

The levels observed in Ho^{166} are all formed by coupling the 523↑ proton to different neutron configurations. The neutron level ordering is identical to that observed in Er^{167}.

The beta decay of 27 h Ho^{166} populates a 1− level at 1826 keV with a log ft = 5.2, which establishes the level configuration as 523↓−633↑ (n) (ref. 33). The long lived Ho^{166} decays entirely to a 6− state at 1785 keV with a log $ft \leqslant 6.7$. Although the decay rate is somewhat retarded for an *au* transition, we assign the 6− state as the $\Sigma = 0$ state of the 633↑+523↓(n) configuration (ref. 34), as no other 6− state is expected at such a low energy. The doublet spacing in the even-even nucleus is 40 keV. This difference is considerably smaller than the spacing observed in other nuclei, although experimentally this is the best established case.

The 1− state at 1663 keV populated by the 27 h Ho^{166} can be interpreted as the intrinsic $K\pi = 0-$ state 404↓−523↑ (p) (the 1− state lying lower than the 0−), but it occurs at a somewhat lower energy than that calculated for the level. The 1460 keV 0+ state is quite naturally interpreted as the $K, K; K+1, K+1$ proton pair excitation.

The decay of the 6− level at 1785 keV populates 7+, 6+, and 5+ states of a $K\pi = 2+$ band. The 4+, 3+ and 2+ levels of this band are then excited by cascade transitions. These latter three levels are well established by the decay of Tm^{166}, in which these levels are strongly excited by cascade transitions. The 7 h Tm^{166} has a measured spin of 2, and two configurations can be assigned this spin, the 2− $\Sigma = 0$ configuration 411↓−523↓, and the 2+ $\Sigma = 1$ configuration 411↓−642↑. A measurement of the magnetic moment should be decisive in establishing the correct assignment†.

The decay scheme of Tm^{166} is complex, and because of the uncertainty in most of the experimental assignments we do not attempt a more detailed analysis of the level scheme.

† See caption of Fig. 10.

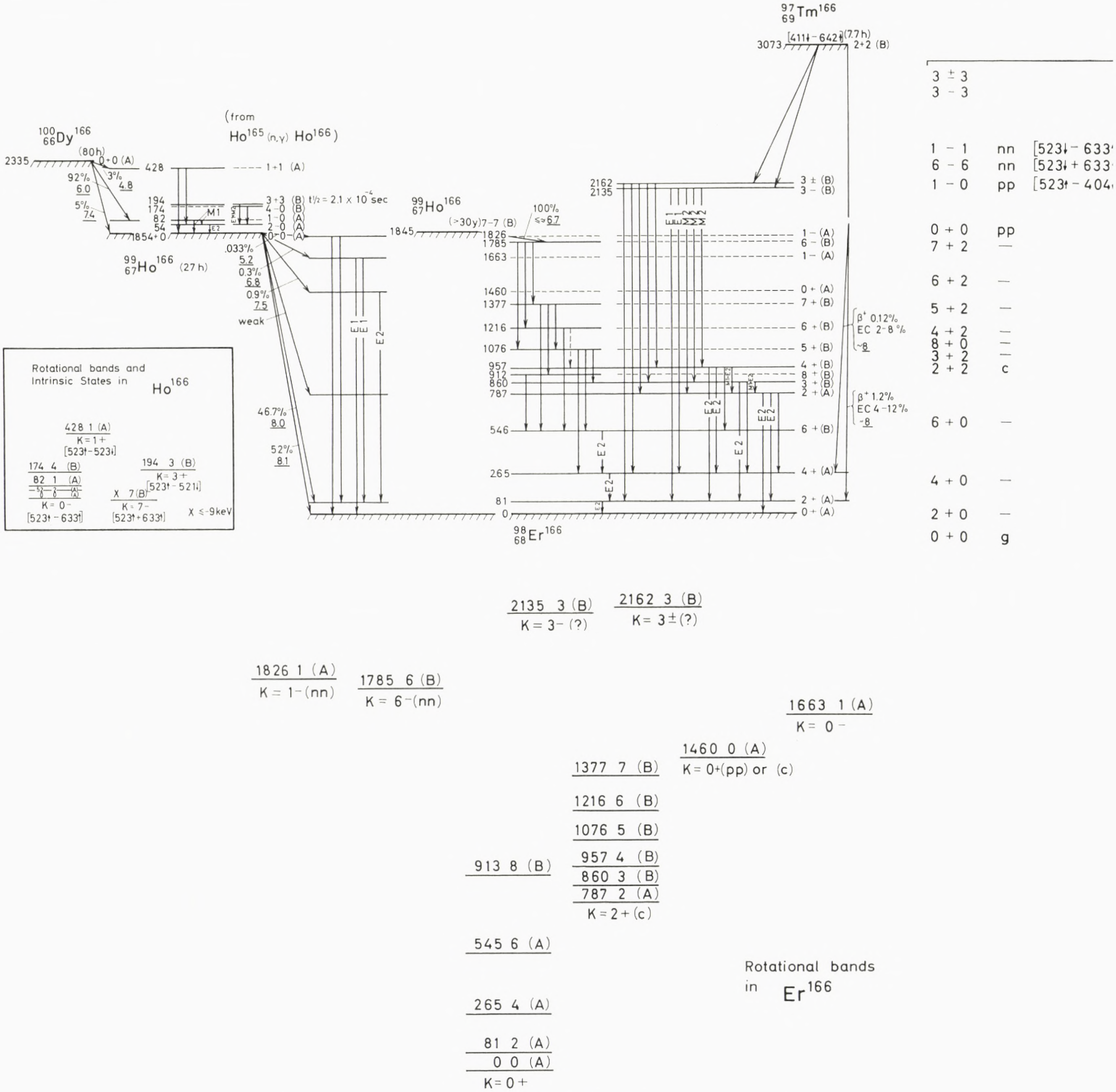

Fig. 10. A = 166.

The decay scheme of Dy^{166} is as reported by R. G. Helmer and S. B. Burson, Phys. Rev. **119**, 788 (1960); Argonne National Laboratory Report ANL 6270 (January 1961, unpublished); J. S. Geiger, R. L. Graham, and G. T. Ewan, Bull. Am. Phys. Soc. **5**, 255 (1960). The additional levels in Ho^{166} have been reported by K. Alexander and V. Bredel, Nuclear Phys. **16**, 152 (1960); I. V. Estulin, A. S. Melioransky, and L. F. Kalinkin, Nuclear Phys. **24**, 118 (1961). The decay of long-lived Ho^{166} has recently been investigated

Ho^{166} (0−0)−1
(7−7)−2

Table III 8 a. $^{98}_{68}Er^{166}$.

Experimental		Theoretical			1†		2†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft	Class	log ft
1−	1.826	523 ↓ − 633 ↑ n	1−1	1.6	au	5.2	..	..
6−	1.785	523 ↓ + 633 ↑ n	6−6	1.6	..	..	au	$\lesssim 6.7$
1−	1.663	523 ↑ − 404 ↓ p	1−0	2.0	ah	6.8	..	..
		(collective)	1−0	..	a?	..	..	..
0+	1.460	proton	0+0	1.7	1 u	7.5	..	..
		(collective)	0+0	..	1?	..	..	..
2+	.781	523 ↓ − 521 ↓ n	2+2	1.7	1*F	large	..	..
		411 ↑ − 411 ↓ p	2+2	1.8	1*F	..	..	..
		(collective)	2+2	..	1*?	..	..	..
2+	.081	ground	2+0	..	1*u	8.0	..	..
0+	0	ground	0+0	..	1 u	8.1	..	..

† 1) 523 ↑ − 633 ↑ 2) 523 ↑ + 633 ↑

A = 168.

The spin coupling rules predict a 3+ ground state for Tm^{168}, the configuration 411 ↓ − 633 ↑. The assignment is supported by the Tm^{168} decay scheme.

Two low-lying 3− levels are predicted in Er^{168}, a proton level 411 ↓ − 523 ↑ at ≈ 1.3 MeV, and a neutron level 633 ↑ − 521 ↓ at ≈ 1.1 MeV, in good agreement with the observed energies of 1095 and 1543 keV. The electron capture transition

† A reanalysis of angular distributions of gamma rays from aligned Ho^{166} nuclei, assuming the long-lived Ho^{166} decay scheme shown, clearly establishes the spin of Ho^{166} as 7, and the spins of the 1785 and 1076 states as 6 and 5, respectively. (Private communication from H. Postma). These levels should therefore be designated (A) in the figure, as should the 2+ Tm^{166} assignment (see below).

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by C. J. Gallagher, Jr., O. B. Nielsen, O. Skilbreid, and A. W. Sunyar, Phys. Rev. (to be published), who review earlier experimental results and report the decay scheme shown. The spin of 0 for 27 h Ho^{166} has been deduced from the atomic beam magnetic resonance data of L. S. Goodman, W. J. Childs, R. Marrus, I. P. K. Lindgren, and Y. Cabezas, Bull. Am. Phys. Soc. **5**, 344 (1960); W. J. Childs and L. S. Goodman, Phys. Rev. **112**, 591 (1961). Negative parity is deduced from its decay to the 2+ rotational level in Er^{166}. The decay energy for the 27 h isomer is as reported by R. L. Graham, J. L. Wolfson, and M. A. Clark, Phys. Rev. **98**, 1173 A (1955). The decay of Ho^{166} to the high lying states of Er^{166} has been reported by P. G. Hansen, K. Wilsky, D. J. Horen, and Lung-Wen Chiao, Nuclear Phys. **24**, 519 (1961). A review of earlier data can be found in this reference and in the NDS. The beta branch to the 2+ level at 787 keV has been observed in Copenhagen (C. J. Gallagher Jr. and O. Skilbreid, unpublished data, 1960). The spin of Tm^{166} has been measured to be 2 (J. C. Walker and D. L. Harris, Phys. Rev. **121**, 224 (1961)). The magnetic moment of Tm^{166} has been measured and clearly establishes the 2+ assignment shown. (J. C. Walker, private communication). Preliminary results on the decay scheme of Tm^{166} have recently been reported by a number of authors (P. Boskma and H. De Waard, Nuclear Phys. **12**, 533 (1959); R. G. Wilson and M. L. Pool, Bull. Am. Phys. Soc. **5**, 155 (1960); K. P. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960); K. Gromov, B. S. Dzhelepov, and V. N. Pokrovski, Izvest. Akad. Nauk. SSSR. Ser. Fiz. **23**, 821 (1959); E. P. Grigoriev, K. Ya. Gromov, and B. S. Dzhelepov, *ibid* **25**, (1961, to be published); J. Zylicz, Y. Preibisz, S. Chojnacki, J. Wotowoski, Yu. Morseev (priv. comm. to A. Bohr and B. Mottelson, Jan. 1961; HHM 61). Only the well-established features of the decay scheme are shown. There is some question whether the total decay energy shown iscorrect.

$^{98}_{68}\mathrm{Er}^{166}$

TABLE III 8b.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{99}_{67}\mathrm{Ho}^{166}$ 0− $p = K$; $n = K+1$ Class	log ft	$^{99}_{67}\mathrm{Ho}^{166}$ 7− $p = K$; $n = K+1$ Class	log ft	$^{97}_{69}\mathrm{Tm}^{166}$ 2+ $p = K+1$; $n = K-1$ Class	log ft
				neutron levels					
$K, K+1$	6−	1.6	1.785	..	..	*au*	≤ 6.7 (5.1)	..	..
	1−	..	1.826	*au*	5.2 (5.1)	..	..	1 *F*	..
$K, K+2$	2+	1.7	..	1**F*	..	..	..	*aF*	large
	3+	..	..	..	..	..	..	*aF*	..
$K-1, K+1$	1+	1.8	..	1 *h*	..	..	..	*a*(2)	..
	6+	..	..	..	..	1 *h*	..	..	..
$K-1, K+2$	3−	1.9	..	..	..	..	..	1*A*(1*u*)	..
	2−	..	..	..	..	..	..	1 *u*	(6.4)
$K, K+3$	5+	2.1	..	..	..	1**F*	..	..	..
	0+	..	..	1 *F*	..	..	..	..	..
$K-2, K+1$	2−	2.1	..	..	..	..	..	1 *F*	..
	5−	..	..	..	..	..	..	..	..
K, K .... / $K+1, K+1$	0+	2.1	..	1 *u*	..	..	..	..	..
$K+1, K+2$	4−	2.2	..	..	..	..	..	1**F*	..
	3−	..	..	..	..	..	..	1 *F*	..
$K-1, K$	5−	2.4	..	..	..	..	..	..	..
	0−	..	..	*aF*	..	..	..	1**h*	..
$K+1, K+3$	1−	2.5	..	*ah*	(6.9)	..	..	1 *F*	..
	6−	..	..	..	..	*ah*	(6.9)	..	..

$K-2 = 521\uparrow$, $K-1 = 642\uparrow$, $K = 523\downarrow$, $K+1 = 633\uparrow$, $K+2 = 521\downarrow$, $K+3 = 512\uparrow$.

State	$K\pi$	Calc.	Exper.	Class	log ft	Class	log ft	Class	log ft
				proton levels					
$K, K+1$	4−	1.3	..	..	..	..	..	1*A*(1*h*)	..
	3−	..	..	..	..	..	..	1 *h*	..
K, K .... / $K+1, K+1$	0+	1.7	1.460	1 *u*	7.5	..	..	..	..
$K-1, K+1$	2+	1.8	..	1**F*	..	..	..	*aA*(*ah*)	..
	1+	..	..	1 *F*	..	..	..	*ah*	..
$K, K+2$	7−	2.0	..	..	..	*ah*	(6.0)	..	..
	0−	..	1.663	*ah*	6.8 (6.0)	..	..	1**F*	..
$K-2, K+1$	2+	2.1	..	1**F*	..	..	..	*ah*	(6.6)
	3+	..	..	..	..	..	..	*aA*(*ah*)	..
$K-3, K+1$	3−	2.2	..	..	..	*aF*	..	1*A*(1*u*)	..
	1−	..	..	*aF*	..	..	..	1 *u*	..
$K-1, K$	2−	2.3	..	..	..	..	..	1 *F*	..
	5−	..	..	..	..	..	..	..	..
$K+1, K+2$	3+	2.4	..	..	..	..	..	*ah*	..
	4+	..	..	..	..	..	..	..	..
$K, K+3$	1−	2.4	..	*ah*	..	..	..	1 *F*	..
	6−	..	..	..	..	*ah*	..	..	..
$K-1, K+2$	5+	2.5	..	..	..	1**F*	..	..	..
	2+	..	..	1**F*	..	..	..	1 *F*	..

$K-3 = 532\uparrow$, $K-2 = 413\downarrow$, $K-1 = 411\uparrow$, $K = 523\uparrow$, $K+1 = 411\downarrow$, $K+2 = 404\downarrow$, $K+3 = 402\uparrow$.

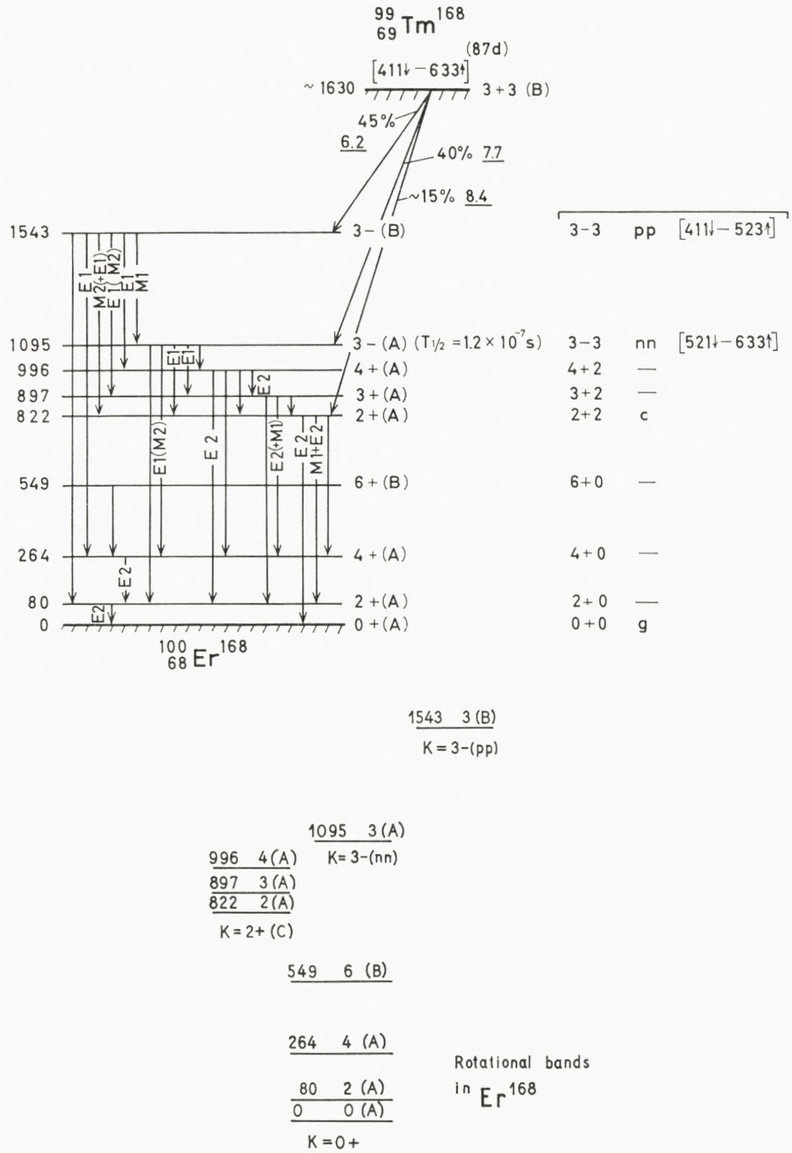


Fig. 11. A = 168.
The decay scheme of Tm^{168} is as reported by K. P. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960). References to earlier work can be found in the NDS.

(log ft = 7.7) to the 1095 keV level is, however, retarded with respect to the same single-particle transition in odd-mass nuclei, where log ft is 6.4.

It is interesting to note that electron capture from the 3+ configuration to all states other than the two 3− states observed is theoretically forbidden. However, because both 3− states have $\Sigma = 1$, it is possible that the 4− $\Sigma = 0$ states of the two

Tm^{168} (3+3)−1 TABLE III 9a. $^{100}_{68}Er^{168}$

Experimental		Theoretical			1†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft*
3−	1.543	411 ↓ − 523 ↑ p	3 − 3	1.3	1 u	6.1 to 7.4
3−	1.095	521 ↓ − 633 ↑ n	3 − 3	1.1	1 u	7.6 to 8.0
2+	.822	521 ↓ − 512 ↑ n	2 + 2	1.6	aF	8.2 to 8.7
..	..	411 ↑ + 411 ↓ p	2 + 2	1.8	aA(2)	..
..	..	(collective)	2 + 2	..	a?	..

† 411 ↓ − 633 ↑
* if $1.65 \leqslant Q_{ec} \leqslant 2.0$.

doublets lie lower in energy than the 3− states, and hence may be populated by weak gamma-ray transitions from them.

The 822, 897, and 996 keV levels form a $K\pi = 2+$ rotational band. All intrinsic 2+ states are expected above ≈ 1.6 MeV, hence the low energy of the state probably indicates collective character.

A = 170.

Tm^{170} has spin 1 and a magnetic moment $|\mu| = 0.25$ nm, in good agreement with what is expected for the configuration 411 ↓ + 521 ↓. Additional support for this assignment is provided by the log $ft = 9.0$ for the beta decay to the Yb^{170} ground

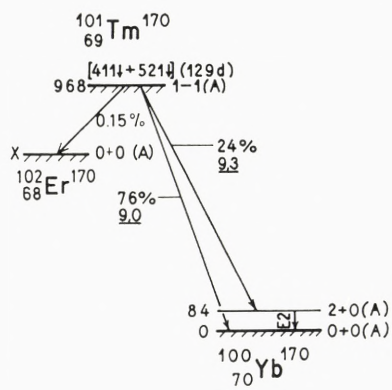


Fig. 12. A = 170.
The decay scheme of Tm^{170} to Yb^{170} is essentially as reported by R. L. GRAHAM, J. L. WOLFSON, and R. E. BELL, Can. J. Phys. **30**, 459 (1952). The 0.15 °/₀ K-capture branch to Er^{170} has been reported by P. P. DAY, Phys. Rev. **102**, 1572 (1956). The spin of Tm^{170} has recently been measured as 1, the magnetic moment $|\mu| = 0.25$ nm. (I. LINDGREN, A. CABEZAS, and W. NIERENBERG, Bull. Am. Phys. Soc. **5**, 273 (1960)). Preliminary investigations of the Lu^{170} decay scheme have been made by several authors (B. HARMATZ, T. H. HANDLEY, and J. W. MIHELICH, Phys. Rev. **119**, 1345 (1960); B. S. DZHELEPOV, I. F. UCHEVATKIN, and S. A. SHESTOPALOVA, Izvest. Akad. Nauk SSSR, Ser. Fiz. **24**, 802 (1960); V. V. TUCHKEVICH, V. A. ROMANOV, and M. G. IODKO, Izvest. Akad. Nauk SSSR, Ser. Fiz. **24**, 1457 (1960)), but are not included here.

$^{100}_{63}Er^{168}$

Table III 9b.

State	$K\pi$	Energy (MeV) Calcul.	Energy (MeV) Exper.	$^{99}_{69}Tm^{168}$ $p=K+1$, Class	$3+$ $n=K$ log ft
neutron levels					
$K, K+1$	$4-$	1.1	..	$1A(1\ u)$	..
	$3-$	..	1.095	$1\ u$	6.0 (6.2)
K, K / $K+1, K+1$	$0+$	1.2	..	..	..
$K, K+2$	$1-$	1.5	..	1^*u	..
	$6-$	..	..	..	..
$K+1, K+2$	$3+$	1.6	..	aF	..
	$2+$	..	..	aF	..
$K-1, K+1$	$2+$	2.0	..	aF	..
	$3+$	..	..	aF	..
$K-1, K$	$6-$	2.1	..	..	..
	$1-$	..	..	1^*h	..
$K-1, K+2$	$5+$	2.2	..	..	..
	$0+$	..	..	..	..
$K-2, K+1$	$3-$	2.2	..	$1\ F$	..
	$2-$	..	..	$1\ F$	..
$K-2, K$	$1+$	2.3	..	..	..
	$6+$	..	..	..	..
$K, K+3$	$7-$	2.3	..	..	..
	$0-$	..	..	..	..

$K-2 = 642\uparrow$, $K-1 = 523\downarrow$, $K = 633\uparrow$, $K+1 = 521\downarrow$, $K+2 = 512\uparrow$, $K+3 = 514\downarrow$

State	$K\pi$	Calcul.	Exper.	Class	log ft
proton levels					
$K, K+1$	$4-$	1.3	..	$1A(1\ u)$	..
	$3-$	..	1.543	$1u$	7.7 (6.5)
K, K / $K+1, K+1$	$0+$	1.7	..	..	..
$K-1, K+1$	$2+$	1.8	..	$aA(2\ h)$	..
	$1+$	..	..	..	..
$K, K+2$	$7-$	2.1	..	..	..
	$0-$	..	..	..	..
$K-2, K+1$	$2+$	2.2	..	ah	..
	$3+$	..	..	$aA(ah)$	..
$K-1, K$	$2-$	2.3	..	$1\ F$	..
	$5-$	..	..	1^*F	..
$K+1, K+2$	$3+$	2.4	..	ah	..
	$4+$	..	..	$aA(ah)$	..

$K-2 = 413\downarrow$, $K-1 = 411\uparrow$, $K = 523\uparrow$, $K+1 = 411\downarrow$, $K+2 = 404\downarrow$.

Tm^{170} (1 − 1) − 1 Table III 10 a. $^{100}_{70}Yb^{170}$

Experimental		Theoretical			1†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft
2 +	.084	ground	2 + 0	–	1 Λ(1 u)	9.3
0 +	0	ground	0 + 0	–	1 Λ(1 u)	9.0

† 411 ↓ + 521 ↓

state, which is classified as 1 Λ(1 u). Because the 1 u transition 521 ↓→411 ↓ is usually observed to have a log ft = 6.4, the Λ-forbiddeness in this case results in a retardation of ≈ 400 in the transition rate.

A = 172.

The ground-state configuration of Tm^{172} is clearly established as 2 − on the basis of its decay to the 0 +, 2 +, and 4 + members of the ground-state bands of Yb^{172}. The log ft = 8.7 for the ground-state transition is consistent with a $1^{*}u$ assignment of the transition. The configuration assignment 411 ↓ − 512 ↑ is therefore unambiguously established.

The 4 − configuration 404 ↓ + 521 ↓ is predicted for Lu^{172}. Recently, an extremely complex decay scheme of Lu^{172} has been proposed by B. Harmatz, T. H. Handley and J. W. Mihelich (35). These authors have assigned 18 excited levels in the spectrum, which they have been able to analyze into 7 rotational bands, 6 based on excited intrinsic states. The electron capture decay of Lu^{172} populates only states with $K\pi$ = 4 +, hence the experimental data are consistent with the assignment of Lu^{172} as 4 −.

Using the rotational band analysis proposed by Harmatz et al., the intrinsic spectrum of Yb^{172} can be interpreted very simply. Two K = 4 + states are predicted in the level spectrum, the proton state 404 ↓ + 411 ↓ and the neutron state 514 ↓ + 521 ↓, and both should be populated by 1 u transitions. We assign the 2194 keV and 2075 keV levels as these configurations, but we are unable to distinguish between them.

Both of the 4 + states have Σ = 1, consequently the Σ = 0 3 + states of the doublets might be expected somewhat lower in energy. Two 3 + intrinsic states have been identified at 1664 and 1702 keV, which we assign as these states. We are again unable to distinguish between the two, but it is not unlikely that the ordering of the configurations will be the same for both the Σ = 0 and Σ = 1 states. We have therefore arbitrarily assumed the ordering shown.

Two other intrinsic states have been identified, a 2 + state at 1468 keV and a 3 + state at 1174 keV. We assign these states as the Σ = 1 and Σ = 0 states, respectively, of the neutron configuration 512 ↑ ± 521 ↓. However, the moment of inertia of the 2 + state is about the same as those of collective 2 + states in this region, so that the interpretation of this level must be regarded as tentative.

With these configuration assignments, the apparent absence of transitions to

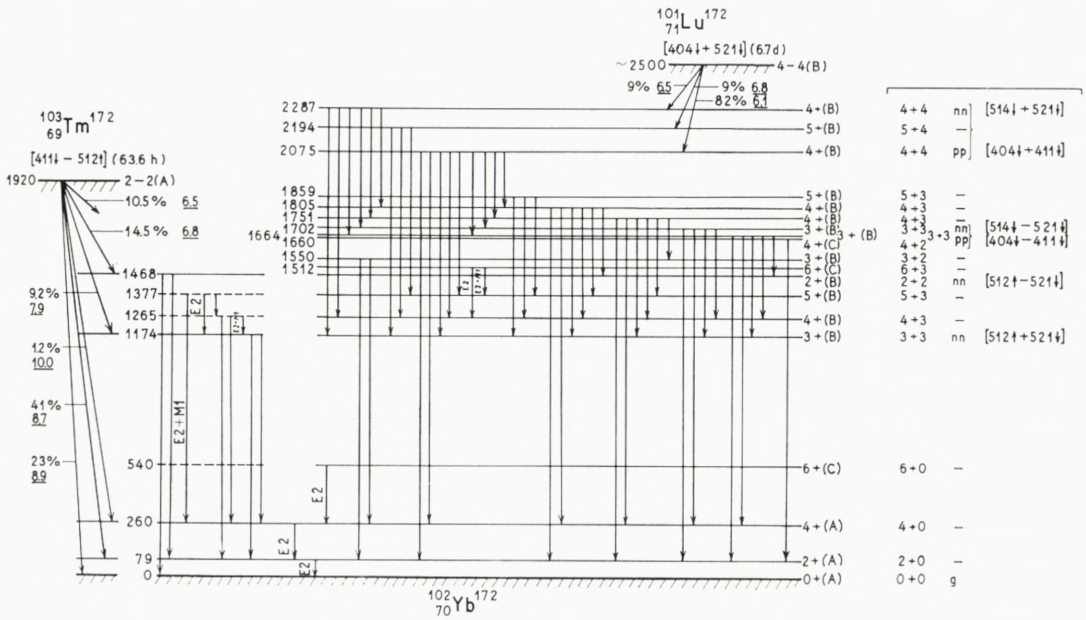
$^{100}_{70}Yb^{170}$ Table III 10b.

State	$K\pi$	Energy (MeV)		$^{101}_{69}Tm^{170}$ 1 – $p = K$; $n = K+1$	
		Calc.	Exper.	Class	log ft
neutron levels					
$K, K+1$	4 –	1.1		..	
	3 –	..		..	
K, K / $K+1, K+1$	0 +	1.2		1 F	
$K, K+2$	1 –	1.5		aF	
	6 –	..		..	
$K-1, K+1$	2 +	2.0		$1 A (1^* h)$	
	3 +	..		$1^* h$	
$K-1, K$	6 –	2.1		..	
	1 –	..		aF	
$K-1, K+2$	5 +	2.2		..	
	0 +	..		1 F	
$K-2, K+1$	3 –	2.2		..	
	2 –	..		$aA(2)$	
$K-2, K$	1 +	2.3		1 F	
	6 +	..		..	
$K, K+3$	7 –	2.3		..	
	0 –	..		aF	

$K-2 = 642\uparrow$, $K-1 = 523\downarrow$, $K = 633\uparrow$, $K+1 = 521\downarrow$, $K+2 = 512\uparrow$, $K+3 = 514\downarrow$.

State	$K\pi$	Calc.	Exper.	Class	log ft
proton levels					
$K, K+1$	3 +	1.4		$1^* A (3)$	
	4 +	..		..	
$K, K+2$	3 +	1.7		$1^* h$	
	2 +	..		$1 A (1^* h)$	
$K, K+3$	5 –	1.8		..	
	4 –	..		..	
K, K / $K+1, K+1$	0 +	~ 1.9		$1 A (1 u)$	
$K-1, K+1$	7 –	2.0		..	
	0 –	..		aF	
$K-1, K$	4 –	2.3		..	
	3 –	..		..	
$K+1, K+2$	6 +	2.3		..	
	1 +	..		1 F	
$K+1, K+3$	8 –	2.4		..	
	1 –	..		1 F	
$K-2, K+1$	5 +	2.5		..	
	2 +	..		1 F	

$K-2 = 411\uparrow$, $K-1 = 523\uparrow$, $K = 411\downarrow$, $K+1 = 404\downarrow$, $K+2 = 402\uparrow$, $K+3 = 514\uparrow$.



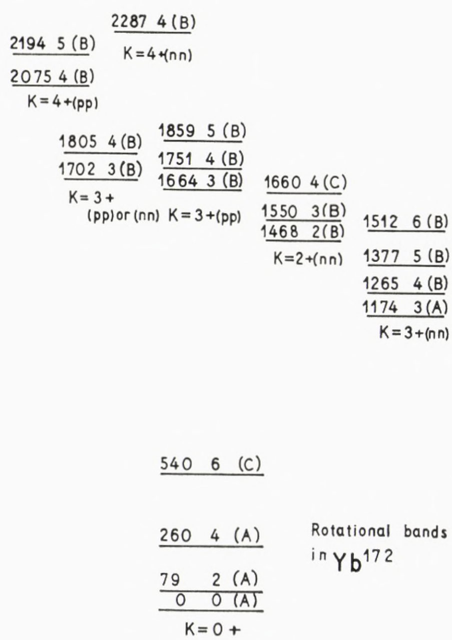


Fig. 13. A = 172.

The decay scheme of Tm^{172} has recently been reported by R. G. Helmer and S. B. Burson, Bull. Am. Phys. Soc. **6**, 72 (1961); Argonne National Laboratory Reprint, ANL-6270, January 1961 (unpublished); P. G. Hansen, O. J. Jensen, and K. Wilsky, Nuclear Phys. **27**, 516 (1961); C. J. Orth and B. J. Dropesky, Phys. Rev. **122**, 1295 (1961). These authors also report data on Er^{172} decay. The Lu^{172} decay scheme has recently been studied by a number of authors (R. G. Wilson and M. L. Pool, Phys. Rev. **118**, 1067 (1960); B. S. Dzhelepov, I. F. Uchevatkin, and S. A. Shestopalova, Izvest. Akad. Nauk SSSR Ser. Fiz. **24**, 802 (1960); V. V. Tuchkevich, V. A. Romanov, and M. G. Iodko, Izvest. Akad. Nauk SSSR, Ser. Fiz. **24**, 1457 (1960); (see also the NDS) but the results shown here are based on the recent results of HHM 61, which are consistent with the results of the above mentioned authors, but more extensive. The analysis into rotational bands was proposed by HHM 61.

Lu^{172} (4−4)−1
Tm^{172} (2−2)−2

TABLE III 11 a. $^{102}_{70}Yb^{172}$

Experimental		Theoretical			1†		2†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft	Class	log ft
4+	2.287	514 ↓ + 521 ↓ n	4+4	1.9	1u	6.5	..	..
4+	2.075	411 ↓ + 404 ↓ p	4+4	1.4	1u	6.1	..	..
3+	1.702	514 ↓ − 521 ↓ n	3+3	1.9	1Λ(1u)	>7.7	..	..
3+	1.664	411 ↓ − 404 ↓ p	3+3	1.4	1Λ(1u)	>7.7	..	..
2+	1.468	512 ↑ − 521 ↓ n	2+2	1.2	1*F	>8.1	1u	6.8
		collective(?)	2+2	..	..	..	1?	..
3+	1.174	512 ↑ + 521 ↓ n	3+3	1.2	1 F	>8.3	1Λ(1u)	7.9
4+	.260	ground	4+0	–	..	..	1*u	10.0
2+	.079	ground	2+0	–	..	..	1*u	8.7
0+	0	ground	0+0	–	..	..	1*u	8.9

† 1) 404 ↓ + 521 ↓ 2) 411 ↓ − 512 ↑

the 1702 and 1664 keV levels is explained as Λ-forbiddenness. Transitions to the 1174 keV level are *ah*, the 404 ↓ (p)→512 ↑ (n) being strongly hindered experimentally. The log ft's for β-decays to the 1174 and 1468 keV levels from Tm^{172} also follow naturally from the above configuration assignment*.

The theoretical analysis of the level spectrum indicates that both Lu^{172} and Tm^{172} should populate additional levels in Yb^{172} (see Table III 11.b). The 1920 keV decay energy of Tm^{172} probably inhibits some of these transitions, but low energy groups in addition to those shown have been observed. We have not included them in the figure because the low resolution employed in these studies would not have been sufficient to resolve the many transitions expected. The decay energy of Lu^{172} is large and allows population of many more states, and it is therefore not surprising that 41 transitions reported by HARMATZ et al. have *not* been assigned in the decay scheme.

A = 174.

Two isomers of Lu^{174} are observed, 1− and 6−. The coupling rules predict the 1− configuration 404 ↓ − 512 ↑ for the Lu^{174} ground state. The 6− isomeric state is most easily described as the $\Sigma = 0$ member of the ground state doublet. The half-life of the 1− state is not known, but it apparently decays only to the ground, the 76 keV 2+ state of the ground state band and a 2− level at 1321 keV. No negative parity proton states are expected as low as 1300 keV in Yb^{174}, but the 2− neutron configuration 624 ↑ − 512 ↑ has a low energy and should be populated by an *ah* transition. However, there is a considerable difference between the calculated and

* Possible intrinsic state assignments based on Tm^{172} decay rates have also been discussed by R. G. HELMER and S. B. BURSON[36].

$^{102}_{70}Yb^{172}$

TABLE III 11 b.

State	$K\pi$	Energy (Mev) Calc.	Energy (Mev) Exper.	$^{103}_{69}Tm^{172}$ 2 − $p = K;\ n = K+1$ Class	log ft	$^{101}_{71}Lu^{172}$ 4 − $p = K+1;\ n = K$ Class	log ft
			neutron levels				
$K, K+1$	3 +	1.1	1.174	$1A(1\ u)$	7.9	$1\ h$	> 8.3
	2 +	. .	1.468	$1\ u$	6.8 (6.6)	$1^*A(1\ h)$	> 8.1
K, K / $K+1, K+1$	0 +	~ 1.3	. .	1^*F	. .	. .	. .
$K-1, K+1$	1 −	1.4	. .	a (2)	. .	. .	. .
	6 −	. .	. .	. .	. .	. .	. .
$K-1, K$	4 −	1.7	. .	. .	. .	ah	. .
	3 −	. .	. .	aF	. .	$aA(ah)$	. .
$K, K+2$	3 +	1.9	1.702	$1\ F$	. .	$1A(1\ u)$	> 7.7
	4 +	. .	2.287	1^*F	. .	$1\ u$	6.5 (6.3)
$K+1, K+2$	6 +	2.0	. .	. .	. .	1^*F	. .
	1 +	. .	. .	1 (3)	. .	. .	. .
$K-1, K+2$	7 −	2.1	. .	. .	. .	. .	. .
	0 −	. .	. .	. .	. .	. .	. .
$K-2, K+1$	5 +	2.3	. .	. .	. .	$1\ F$	. .
	0 +	. .	. .	1^*h	. .	. .	. .
$K, K+3$	5 −	2.4	. .	. .	. .	ah	. .
	4 −	. .	. .	. .	. .	$aA(ah)$	. .

$K-2 = 523\downarrow,\ \ K-1 = 633\uparrow,\ \ K = 521\downarrow,\ \ K+1 = 512\uparrow,\ \ K+2 = 514\downarrow,\ \ K+3 = 624\uparrow.$

State	$K\pi$	Calc.	Exper.	Class	log ft	Class	log ft
			proton levels				
$K, K+1$	3 +	1.4	1.664	$1\ h$	(8.7)	$1A(1\ u)$	> 7.7
	4 +	. .	2.075	$1A(1\ h)$	. .	$1\ u$	6.1 (6.6)
$K, K+2$	3 +	1.7	. .	$1A(1\ u)$	. .	$1\ F$	. .
	2 +	. .	. .	$1\ u$	(6.0)	1^*F	. .
$K, K+3$	5 −	1.8	. .	. .	. .	aF	. .
	4 −	. .	. .	. .	. .	aF	. .
K, K / $K+1, K+1$	0 +	~ 1.9	. .	1^*u	. .	. .	. .
$K-1, K+1$	7 −	2.0	. .	. .	. .	. .	. .
	0 −	. .	. .	. .	. .	. .	. .
$K-1, K$	4 −	2.3	. .	. .	. .	aF	. .
	3 −	. .	. .	ah	. .	aF	. .
$K+1, K+2$	6 +	2.3	. .	. .	. .	. .	. .
	1 +	. .	. .	$1\ F$	. .	. .	. .
$K+1, K+3$	8 −	2.4	. .	. .	. .	. .	. .
	1 −	. .	. .	aF	. .	. .	. .
$K-2, K+1$	5 +	2.5	. .	. .	. .	$1\ u$	. .
	2 +	. .	. .	$1\ F$	. .	. .	. .

$K-2 = 411\uparrow,\ \ K-1 = 523\uparrow,\ \ K = 411\downarrow,\ \ K+1 = 404\downarrow,\ \ K+2 = 402\uparrow,\ \ K+3 = 514\uparrow.$

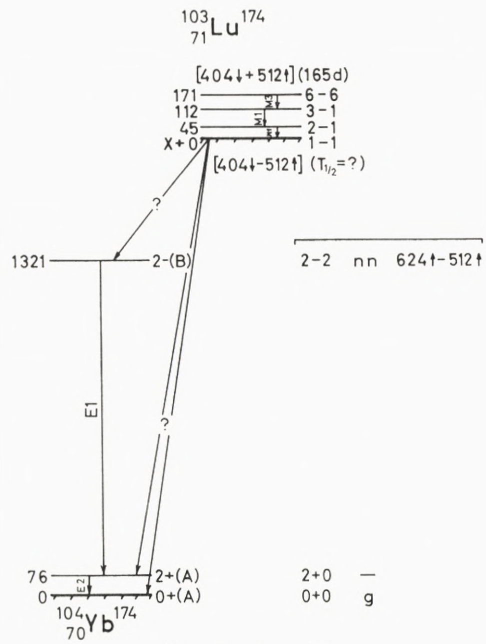


Fig. 14. A = 174.

The levels in Lu^{174} are as reported by B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **119**, 1345 (1960); for earlier results see SHS, the NDS. The decay scheme of the Lu^{174} isomers is as reported by these authors. The level at 1321 keV has recently been established as 2− (H. J. Prask, F. G. Funk, and J. W. Mihelich, priv. comm., May 1961; J. Borggreen, P. Jastram, M. Jørgensen, and O. B. Nielsen, priv. comm., May 1961).

Lu^{174} (1−1)−1 Table III 12 a. $^{104}_{70}$Yb^{174}

Experimental		Theoretical			1†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft
2−	1.321	624 ↑ − 512 ↑ n	2 − 2	2.3	*ah*	–
2+	.076	ground	2 + 0	–	*ah*	–
0+	0	ground	0 + 0	–	*ah*	–

† 404 ↓ − 512 ↑

observed energy for this state, which may reflect the fact that the $N = 104$ level spectrum has been calculated only for the first group of level parameters, but actually $N = 104$ is the neutron number at which the transition from the first to the second group occurs. For this case some deviation might be expected.

A = 176.

The measured spin of Lu^{176} is 7, in agreement with the predicted configuration 404 ↓ + 514 ↓. The magnetic moment of the isomer calculated directly from the Nilsson wave functions is not in good agreement with the measured value, but if the

$^{104}_{70}Yb^{174}$

Table III 12 b.

State	$K\pi$	Energy (MeV)		$^{103}_{71}Lu^{174}$ 1 − $p = K+1$; $n = K$	
		Calc.	Exper.	Class	log ft
neutron levels					
$K, K+1$	6 +	1.6	..	..	..
	1 +	..	..	1 u	..
$K-1, K+1$	3 +	2.0	..	1*F	..
	4 +	..	..	..	..
K, K / $K+1, K+1$	0 +	~ 2.2	..	1 h	..
$K, K+2$	2 −	2.3	1.321	ah	..
	7 −	..	..	..	..
$K-2, K+1$	7 −	2.3	..	..	..
	0 −	..	..	aF	..
$K, K+3$	2 +	2.4	..	1(3)	..
	3 +	..	..	1*A(3)	..
$K-1, K+2$	5 −	2.6	..	..	..
	4 −	..	..	..	..
$K-1, K$	3 +	2.6	..	1*A(3)	..
	2 +	..	..	1(3)	..
$K+1, K+2$	8 −	2.7	..	..	..
	1 −	..	..	aF	..

$K-2 = 633\uparrow$, $K-1 = 521\downarrow$, $K = 512\uparrow$, $K+1 = 514\downarrow$, $K+2 = 624\uparrow$, $K+3 = 510\uparrow$.

State	$K\pi$	Calc.	Exper.	Class	log ft
proton levels					
$K, K+1$	3 +	1.4	..	..	..
	4 +	..	..	..	..
$K, K+2$	3 +	1.7	..	1*F	..
	2 +	..	..	1 F	..
$K, K+3$	5 −	1.8	..	..	..
	4 −	..	..	..	..
K, K / $K+1, K+1$	0 +	~ 1.9	..	1 F	..
$K-1, K+1$	7 −	2.0	..	..	..
	0 −	..	..	ah	..
$K-1, K$	4 −	2.3	..	..	..
	3 −	..	..	..	..
$K+1, K+2$	6 +	2.3	..	..	..
	1 +	..	..	1 u	..
$K+1, K+3$	8 −	2.4	..	..	..
	1 −	..	..	a(2)	..
$K-2, K+1$	5 +	2.5	..	..	..
	2 +	..	..	1 u	..

$K-2 = 411\uparrow$, $K-1 = 523\uparrow$, $K = 411\downarrow$, $K+1 = 404\downarrow$, $K+2 = 402\uparrow$, $K+3 = 514\uparrow$.

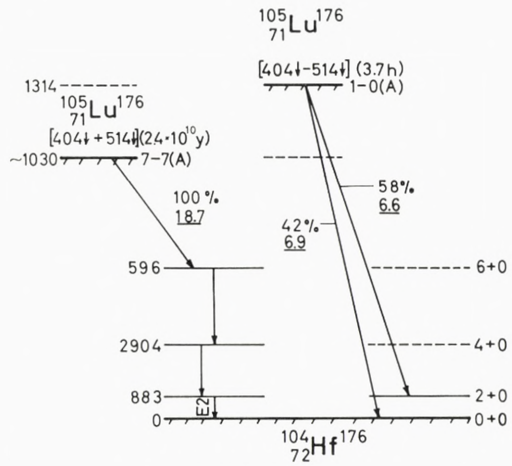


Fig. 15. A = 176.

The spin of 7 of long lived Lu^{176} is based on atomic spectroscopy and Coulomb excitation results. The magnetic moment of +2.8 is determined from atomic spectroscopic measurements. The decay scheme has been checked by several authors (see SHS, the NDS for references). The spin of 3.7 *h* Lu^{176} has recently been measured as 1 (M. B. White, S. S. Alpert, and E. Lipworth, Bull. Am. Phys. Soc. **5**, 273 (1960)). The $K = 0-$ quantum number assignment is based on the recent measurements of the beta branching of this isomer to the 0+ and 2+ levels of Hf^{176}. (I. Řezánka, J. Frána, J. Adam, and L. K. Peker, Izvest. Akad. Nauk SSSR, Ser. Fiz. **26**, 127 (1961)). The beta endpoint energies are also taken from this work. Preliminary studies of Ta^{176} decay (J. O. Rasmussen and D. A. Shirley, University of California Radiation Laboratory Report UCRL-8618 (1959) (unpublished); B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **119**, 1345 (1960)) indicate that the decay scheme is very complex.

Lu^{176} (1−0)−1
(7−7)−2

Table III 13 a. $^{104}_{72}$Hf^{176}

Experimental		Theoretical			1†		2†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft	Class	log ft
6+	.596	ground	6+0	–	..	..	$1K(1u)$	18.7
2+	.088	ground	2+0	–	1u	6.6	..	..
	0	ground	0+0	–	1u	6.9	..	..

† 1) 404 ↓ − 514 ↓ 2) 404 ↓ + 514 ↓

empirical magnetic moment of the 404 ↓ proton is substituted for the calculated 404 ↓ magnetic moment good agreement is obtained[18]. The beta decay of long-lived Lu^{176} is the classical example of K-forbidden beta decay.

Two possible spin 1 isomeric states are possible for the 3.7 h Lu^{176}, the $\Sigma = 0$ doublet state 404 ↓ − 514 ↓ and the $\Sigma = 1$ configuration 404 ↓ − 624 ↑. The recent measurements of the beta branching ratio for the decay of this isomer favour the $K\pi = 0-$ assignment.

$^{104}_{72}Hf^{176}$

TABLE III 13 b.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{105}_{71}Lu^{176}$ 1 − 0, $p = K$; $n = K+1$ Class	$^{105}_{71}Lu^{176}$ 1 − 0, $p = K$; $n = K+1$ log ft	$^{105}_{71}Lu^{176}$ 7 −, $p = K$; $n = K+1$ Class	$^{105}_{71}Lu^{176}$ 7 −, $p = K$; $n = K+1$ log ft
neutron levels							
$K, K+1$	6 +	1.6	..	..	..	1 h	..
	1 +	..	..	1 h	..	..	..
$K-1, K+1$	3 +	2.0	..	..	..	..	..
	4 +	..	..	..	..	..	..
K, K / $K+1, K+1$	0 +	~ 2.2	..	1 F	..	..	..
$K, K+2$	2 −	2.3	..	aF	..	..	..
	7 −	..	..	..	..	aF	..
$K-2, K+1$	7 −	2.3	..	..	..	ah	..
	0 −	..	..	ah	..	..	..
$K, K+3$	2 −	..	..	aF	..	..	..
	3 −	2.4	..	..	..	..	..
$K-1, K-2$	5 −	2.6	..	..	..	..	..
	4 −	..	..	..	..	..	..
$K-1, K$	3 +	2.6	..	1*F	..	..	..
	2 +	..	..	1 F	..	..	..
$K+1, K+2$	8 −	2.7	..	..	..	ah	..
	1 −	..	..	ah	..	..	..

$K-2 = 633\uparrow$, $K-1 = 521\downarrow$, $K = 512\uparrow$, $K+1 = 514\downarrow$, $K+2 = 624\uparrow$, $K+3 = 510\uparrow$.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	$^{105}_{71}Lu^{176}$ 1 − 0, $p = K$; $n = K+1$ Class	$^{105}_{71}Lu^{176}$ 1 − 0, $p = K$; $n = K+1$ log ft	$^{105}_{71}Lu^{176}$ 7 −, $p = K$; $n = K+1$ Class	$^{105}_{71}Lu^{176}$ 7 −, $p = K$; $n = K+1$ log ft
proton levels							
$K, K+1$	8 −	1.0	..	..	..	au	(4.7)
	1 −	..	..	au	..	..	..
K, K / $K+1, K+1$	0 +	~ 1.2	..	1 u	..	..	..
$K-1, K+1$	5 −	1.7	..	..	..	..	..
	4 −	..	..	..	..	..	..
$K, K+2$	6 +	1.8	..	..	..	1 h	..
	1 +	..	..	1 h	..	..	..
$K-1, K$	3 +	1.8	..	1*(3)	..	..	..
	4 +	..	..	..	..	..	..
$K+1, K+2$	2 −	1.9	..	aF	..	..	..
	7 −	..	..	..	..	aF	..
$K-2, K+1$	1 +	2.2	..	1 F	..	..	..
	8 +	..	..	..	..	1 F	..
$K-2, K$	7 −	2.3	..	..	..	ah	..
	0 −	..	..	ah	..	..	..

$K-2 = 523\uparrow$, $K-1 = 411\downarrow$, $K = 404\downarrow$, $K+1 = 514\uparrow$, $K+2 = 402\uparrow$.

A = 178.

Two isomers of Ta^{178} are known. The 9.3 m isomer decays by transitions with log *ft* = 4.6 to the 0+ and 2+ levels in Hf^{178}, clearly indicating the 1+ configuration 514 ↑ − 514 ↓ for the isomer. The 2.1 h isomer decays by a transition with log *ft* = 4.9 to an 8− level at 1148 keV in Hf^{178}. The *au* transition to this 8− state and the similarity in energy to the 1142 keV 5.5 h 8− isomer in Hf^{180} suggest the assignment of the 8− state as the proton configuration 404 ↓ + 514 ↑ and the Ta^{178} isomer as the 7− configuration 404 ↓ + 514 ↓. The latter state is predicted as the Ta^{178} ground state by the spin coupling rules, hence it has been drawn below the 1+ state in the figure.

The energy of the 8− state at 1480 keV is in excellent agreement with the calculated energy of the neutron configuration 624 ↑ + 514 ↓. The population of this state by an electron capture branch with a log *ft* = 4.9 is, however, seriously inconsistent with the interpretation of the state as a two-neutron state, and suggests that the state contains a large amplitude of the 1148 keV two-proton state. The *M*1 transition between them is then consistent with this interpretation.

The 1+ Ta^{178} isomer populates two 0+ levels at 1197 and 1440 keV. The similarity in energy of these two levels to the 8− levels suggests their assignment as proton and neutron pair excitations, respectively. The calculated energies of the lowest-lying pair excitations are in excellent agreement with this interpretation.

The 1+ state at 1430 keV is difficult to explain, because it is populated by an *au* electron-capture transition, and we can give no simple explanation for this fast transition within the framework we discuss. Furthermore, the energy of the state is somewhat lower than the energies of the 1+ states expected in the spectrum. These isomers have previously been discussed by GALLAGHER and NIELSEN(37).

Ta^{178} (7−7)−1
(1+1)−2

TABLE III 14 a.

$^{106}_{72}Hf^{178}$

Experimental		Theoretical			1†		2†	
Iπ	*E*	Final configuration	IπK	*E*	Class	log *ft*	Class	log *ft*
8−	1.480	624 ↑ + 514 ↓ *n*	8−8	1.5	*ah*	4.9	..	..
0+	1.440	neutron	0+0	1.7	..	..	*au*	4.7
1+	1.430	512 ↑ − 514 ↓ *n*	1+1	2.0	..	..	*a*(2)	≈ 4.4
		404 ↓ − 402 ↑ *p*	1+1	1.9	..	..	*aF*	..
0+	1.197	proton	0+0	1.2	..	..	*aF*	5.1
8−	1.148	404 ↓ + 514 ↑ *p*	8−8	1.0	*au*	4.9	..	..
2+	.093	ground	2+0	—	..	..	*au*	4.8
0+	0	ground	0+0	—	..	..	*au*	4.6

† 1) 404 ↓ + 514 ↓ 2) 514 ↑ − 514 ↓

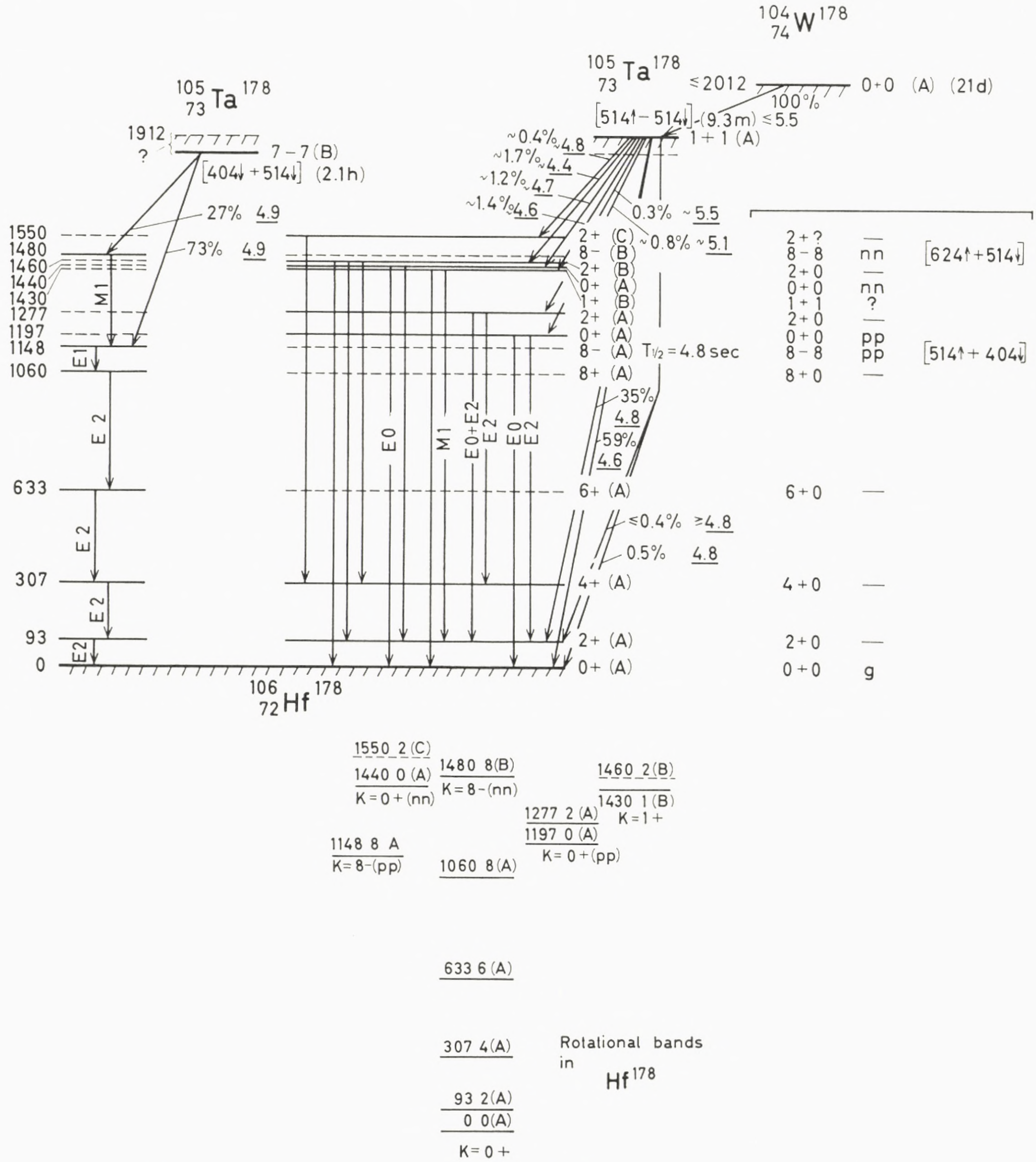


Fig. 16. A = 178.

The decay scheme of 9.3 min. Ta^{178} has recently been studied by C. J. Gallagher, Jr., H. L. Nielsen, and O. B. Nielsen, Phys. Rev. **123**, 1590 (1961); J. Borggreen, U. Bertelsen, and O. Nathan, Phys. Rev. **123**, 564 (1961). Previous experimental studies are reviewed in the former work. The decay scheme of 2.1 *h* Ta^{178} was first reported by F. F. Felber, F. S. Stephens, and F. Asaro, J. Inorg. Nuclear Chem. **7**, 153 (1958). The M1 assignment of the 331.7 keV transition is as proposed by B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **119**, 1345 (1960). The spin 8 of the 1148 keV level has recently been established by M. Deutsch and R. W. Bauer, Nuclear Phys. **21**, 128 (1960). The analysis of the level scheme has previously been discussed (C. J. Gallagher, Jr., and H. L. Nielsen, Phys. Rev. (to be published)). The estimate of the total decay energy of W^{178} to 9.3 *m* Ta^{178} is based on a measurement of an upper limit on the *K*/Total-capture ratio of W^{178} (C. J. Gallagher, Jr., and H. L. Nielsen, unpublished data (1960). Other data on W^{178} decay are as reported in SHS.

$^{106}_{72}Hf^{178}$

TABLE III 14b.

State	$K\pi$	Energy (MeV)		$^{105}_{73}Ta^{178}$ 7 − $p = K+1$; $n = K$		$^{105}_{73}Ta^{178}$ 1 + $p = K$; $n = K$	
		Calc.	Exper.	Class	log *ft*	Class	log *ft*
neutron levels							
$K, K+1$	8 −	1.5	1.480	*ah*	4.9 (6.9)	..	..
	1 −	..	..	..	..	1 *u*	..
K, K; $K+1, K+1$	0 +	1.7	1.440	..	..	*au*	≈ 4.6
$K-1, K+1$	2 −	1.7	..	..	..	1 *F*	..
	7 −	..	..	*aF*	..	..	..
$K, K+2$	4 +	1.9	..	..	..	..	..
	3 +	..	..	..	..	..	..
$K-1, K$	6 +	2.0	..	1 *h*	..	..	..
	1 +	..	..	..	..	*ah*	..
$K, K+3$	2 +	2.3	..	..	..	*a*(2)	..
	5 +	..	..	1**h*	..	..	..
$K-2, K$	3 +	2.3	..	..	..	..	..
	4 +	..	..	..	..	..	..

$K-2 = 521\downarrow$, $K-1 = 512\uparrow$, $K = 514\downarrow$, $K+1 = 624\uparrow$, $K+2 = 510\uparrow$, $K+3 = 512\downarrow$.

State	$K\pi$	Calc.	Exper.	Class	log *ft*	Class	log *ft*
proton levels							
$K, K+1$	8 −	1.0	1.148	*au*	4.9 (4.7)	..	..
	1 −	..	..	..	..	1 *u*	..
K, K; $K+1, K+1$	0 +	1.2	1.197	..	..	*aF*	≈ 5.1
$K-1, K+1$	3 +	1.7	..	..	..	..	..
	4 +	..	..	..	..	..	..
$K, K+2$	2 −	1.8	..	..	..	1 *h*	..
	7 −	..	..	*aF*	..	..	..
$K-1, K$	5 −	1.8	..	..	..	..	..
	4 −	..	..	..	..	..	..
$K+1, K+2$	6 +	1.9	..	1 *h*	..	..	..
	1 +	..	..	..	..	*aF*	..
$K-2, K+1$	7 −	2.2	..	*ah*	..	..	..
	0 −	..	..	..	..	1 *F*	..
$K-2, K$	1 +	2.3	..	..	..	*ah*	..
	8 +	..	..	1 *F*	..	..	..

$K-2 = 523\uparrow$, $K-1 = 411\downarrow$, $K = 514\uparrow$, $K+1 = 404\downarrow$, $K+2 = 402\uparrow$.

A = 180.

The isomeric state in Hf^{180}, in analogy to that in Hf^{178}, is assigned as the proton configuration $514\uparrow + 404\downarrow$.

Two isomers of Ta^{180} are known. The spin coupling rules predict a 1 + configuration $404\downarrow - 624\uparrow$ as the ground state, a 9 − configuration $514\uparrow + 624\uparrow$ as a low

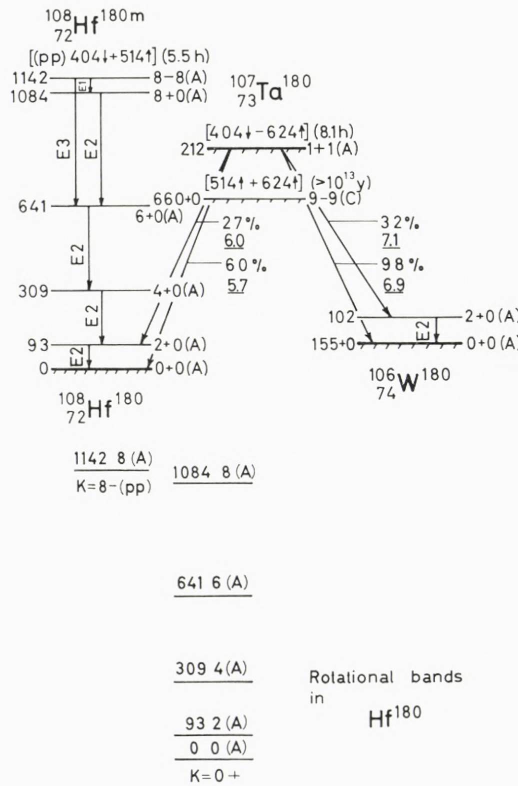


Fig. 17. A = 180.

The decay of 5.5 *h* Hf^{180} is as reported in SHS, the NDS. The spin 8 of the 1142 keV level has recently been established by M. Deutsch and R. W. Bauer, Nuclear Phys. **21**, 128 (1960). Limits on the half-life of the long-lived Ta^{180} using different methods have been set by P. Eberhardt, J. Geiss, and C. Lang, Z. Naturforsch. **10a**, 796 (1955); E. R. Bauminger and S. G. Cohen, Phys. Rev. **110**, 953 (1958). The beta and electron capture branching ratios of 8.15 hour Ta^{180} have been measured by C. J. Gallagher, Jr., M. Jørgensen, and O. Skilbreid, Nuclear Phys. (to be published), but the decay scheme of the isomer is essentially as proposed by H. N. Brown, W. L. Bendel, F. S. Store, and R. A. Becker, Phys. Rev. **84**, 292 (1951). The relative ordering of the two isomers is based on the Ta^{181} (γ, n) Ta^{180} reaction studies of K. N. Geller, J. Halpern, and E. G. Muirhead, Phys. Rev. **118**, 1302 (1960). The E.C. decay energy is based on a recent adjustment of mass values in the rare earth region (A. H. Wapstra, priv. comm. May 1961).

Ta^{180} (1+1)−1
(9−9)−2

Table III 15.

$^{108}_{72}Hf^{180}$ $^{106}_{74}W^{180}$

Experimental		Theoretical			1†		2†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft*	Class	log ft
2+	.102	ground (W^{180})	2+0	–	*ah*	7.1	..	..
0+	0	ground (W^{180})	0+0	–	*ah*	6.9	..	..
2+	.093	ground (Hf^{180})	2+0	–	*ah*	6.2	..	..
0+	0	ground (Hf^{180})	0+0	–	*ah*	5.9	..	..

† 1) 404 ↓ − 624 ↑ 2) 514 ↑ + 624 ↑
* assuming Q_{EC} = .872.

lying isomer. The results of Ta^{181} (γ, n) Ta^{180} reactions, however, seem to indicate that the high spin isomer actually lies below the $1+$ state. On the basis of measurements of the $\beta-$ and electron capture branching ratios of the 8.15 h isomer of Ta^{180}, its $K\pi = 1+$ assignment seems well established.

A = 182.

The beta decay of Ta^{182} populates $2+$, $2-$, $3-$, and $4-$ states in W^{182}, suggesting $3-$ as the most probable configuration for the ground state of Ta^{182}, in agreement with the $3-$ configuration predicted by the coupling rules.

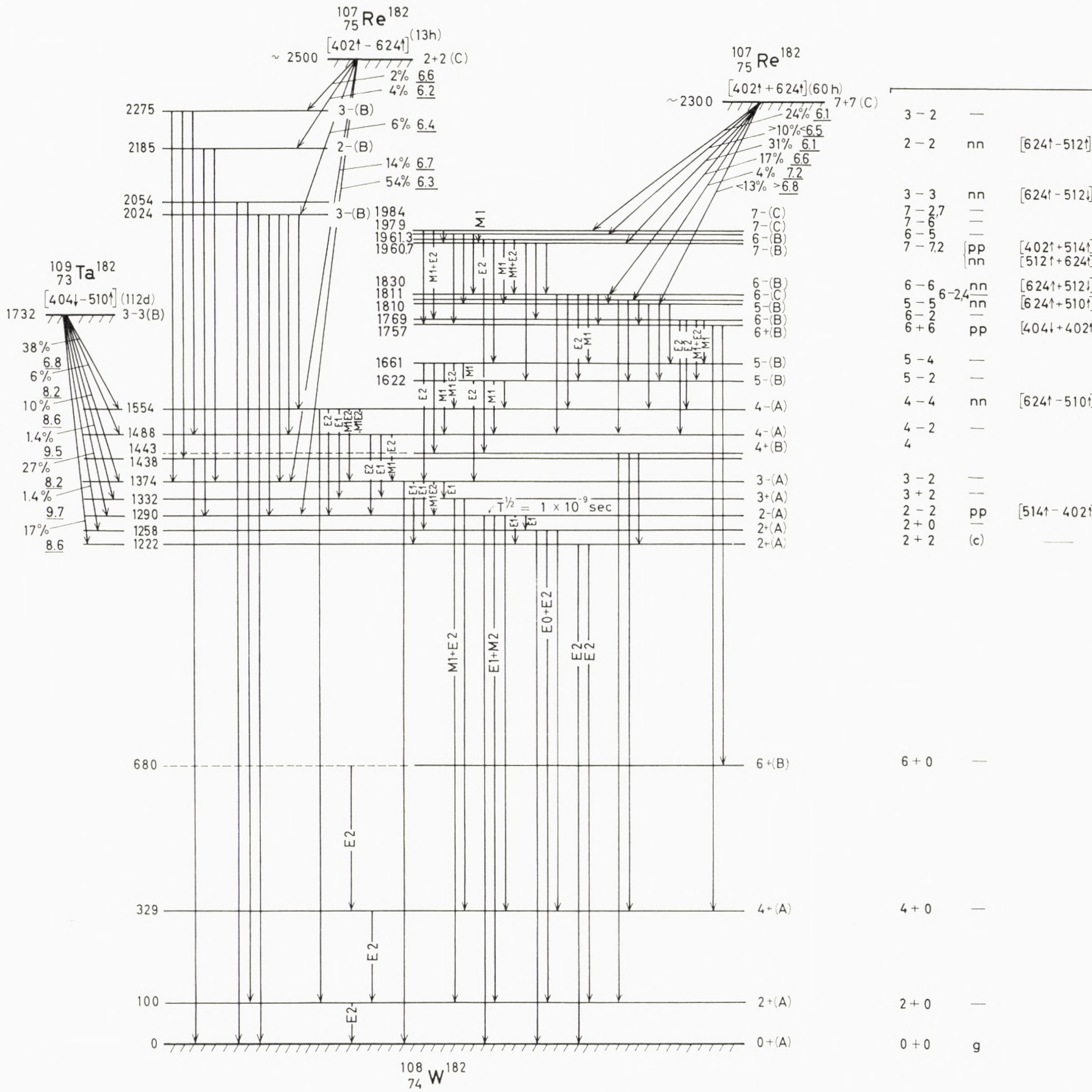
Two Re^{182} isomers are observed. On the basis of the decay schemes of the isomers a $2\pm$ or $3\pm$ assignment is possible for the 13 *h* isomer, and $7\pm$ is the most probable assignment for the 60 h isomer. A $7+$ configuration, $402\uparrow + 624\uparrow$, is predicted as the Re^{182} ground state. Two possible configurations seem equally probable for the 13 h isomer if the energy level systematics in odd-mass nuclei are considered, either the $2+$ $\Sigma = 0$ state of the ground state doublet, or the $\Sigma = 1$ $3-$ neutron excitation state $402\uparrow + 510\uparrow$.

The W^{182} level spectrum determined from the decay of the 3 nucleides is extremely complex, but the apparent complexity is reduced considerably by analysis into rotational bands. Our tentative analysis is shown in Fig. 18.

Beginning with the decay of Ta^{182}, the 1290 keV state appears to be the base state of a $K = 2-$ band, to which belong the 1374 and 1488 keV levels. The 1554 keV state is assigned as the base state of a $K = 4-$ band. The lowest lying intrinsic proton state is the $2-$ configuration $514\uparrow - 402\uparrow$, and the lowest lying neutron state is the $4-$ configuration $624\uparrow - 510\uparrow$. Beta decay from $3-$ Ta^{182} to the $2-$ state is non-overlap forbidden, and the transitions to this state are strongly retarded (but somewhat less than might be expected). Beta decay to the $4-$ state at 1554 keV is *ah*, and the log ft = 6.8 observed for the transition is equal within experimental error to the transition rate for the $404\downarrow(p)\rightarrow 624\downarrow(n)$ transition observed in odd-mass nuclei. The $2+$ state at 1222 seems best described as a collective state. The beta transition to this state is strongly retarded. We are uncertain whether the $K = 0+$ state (unobserved) on which the 1258 keV $2+$ state is based should be assigned as a collective state or as a pair excitation.

The decay of 13 h Re^{182} populates predominantly the $K\pi = 2-$ rotational band at 1290 keV. Decays to this band from the $3-$ state $402\uparrow + 510\uparrow$ are strongly forbidden, whereas transitions from the $2+$ state $402\uparrow - 624\uparrow$ are *ah*, with log ft = 6.8. The 13 h Re^{182} isomer therefore seems best assigned as the $\Sigma = 0$ state of the Re^{182} ground state doublet. A $2-$ state at 2185 keV and a $3-$ state at 2024 keV are also populated by $2+$ Re^{182}. On the basis of the decay ratio to these states, assignments of $624\uparrow - 512\uparrow$ and $624\uparrow - 512\downarrow$, respectively, to these states seem reasonable.

The decay of the $7+$ isomer populates predominantly $6-$ and $7-$ states. A definite analysis of these states is difficult, and the analysis shown must be considered



$^{107}_{75}Re^{182}$
[402↑ - 624↑] (13h)
~ 2500
2+2 (C)
$^{107}_{75}Re^{182}$
[402↑ + 624↑] (60 h)
~2300
7+7 (C)
$^{109}_{73}Ta^{182}$
[404↓-510↑] (112d)
1732
3-3(B)
$^{108}_{74}W^{182}$

Fig. 18. A = 182.

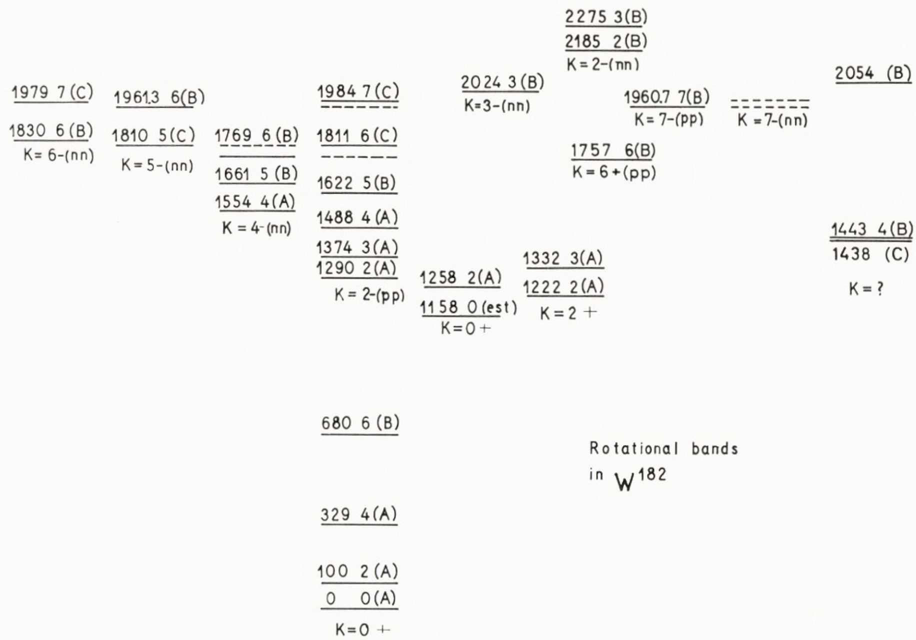


Fig. 18. A = 182.

The decay scheme of Ta^{182} has been extensively studied. The decay scheme shown is essentially that proposed by J. J. Murray, F. Boehm, P. Marmier, and J. W. M. Du Mond, Phys. Rev. **97**, 1007 (1955). It should be noted that the primary beta branchings reported by these authors have been questioned, but the values shown are theirs. (Other work which supports the decay scheme and established the level spin is reported in SHS and the NDS). The 1258 level has recently been reassigned by V. S. Grozdev, L. I. Rusinov, and Yu L. Khazov, Izvest. Akad. Nauk SSSR. Ser. Fiz. **24**, 1444 (1960) from data on Ta^{182} decay and independently by HHM 61 from data on 13 *h* Re^{182} decay. The decay of the 16 m Ta^{182} isomer has recently been studied in some detail (A. W. Sunyar and P. Axel, Phys. Rev. **121**, 1158 (1961)). However, because it is not as yet clear how the levels reported in this work fit into the Ta^{182} level spectrum, we have not included these results in the figure. The levels in W^{182} populated by 13 *h* Re^{182} are as proposed by C. J. Gallagher, Jr., J. O. Newton, and V. S. Shirley, Phys. Rev. **113**, 1298 (1959), with additional levels at ≈ 2 Mev as proposed by HHM 61. The electron capture branching ratios reported by HHM 61 are shown. The electron capture decay of 60 hour Re^{182} has been studied by C. J. Gallagher, Jr., and J. O. Rasmussen, Phys. Rev. **112**, 1730 (1958) and HHM 61. Transitions supporting several of the levels proposed in the former work have been reassigned by the latter authors on the basis of more detailed results. The electron capture branching ratios are as reported by HHM 61.

tentative at least for the 8 states beginning with the 6 – states at 1769 keV. However, the qualitative prediction of many 6 – and 7 – intrinsic states seems well borne out. On the basis of the observed branching, we tentatively assign the 1810 keV 5 – state as the neutron state $624\uparrow + 510\uparrow$, and the 6 – 1830 keV state as the neutron state $624\uparrow + 512\downarrow$. There seem to be at least three 7 – states in the observed spectrum, whereas theoretically, in addition to the two intrinsic 7 – states expected, the neutron state $624\uparrow + 512\uparrow$ and the proton state $514\uparrow + 402\uparrow$, many 7 – rotational states based on the lower-lying negative parity states should also be present.

We have assigned the 1757 keV level as the $K\pi = 6+$ proton state $404\downarrow + 402\uparrow$. This state should be populated directly by electron capture, and although Harmatz et al.[35] indicate no primary capture, the de-exciting gamma-ray intensity seems

$Ta^{182}(3-3)-1$
$Re^{182}(7+7)-2$
$(2+2)-3$

TABLE III 16 a. $^{108}_{74}W^{182}$

Experimental		Theoretical			1†		2†		3†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft	Class	log ft	Class	log ft
2−	2.185	$624\uparrow-512\uparrow$ n	2−2	2.3	..	..	..	..	1 u	6.2
3−	2.024	$624\uparrow-512\downarrow$ n	3−3	1.9	..	..	..	..	1 u	6.4
7−	1.984	rot. state 1.290	7−2	..	..	..	1 K(1 u)	6.1	..	..
..	..	rot. state 1.554	7−4	..	..	..	1 K(1*h)	..	..	..
..	..	rot. state 1.830	7−6	..	..	..	1 u	..	..	..
..	..	$624\uparrow+512\uparrow$ n	7−7	2.3	..	..	1 u	..	..	..
..	..	$514\uparrow+402\uparrow$ p	7−7	1.3	..	..	1 u	..	..	..
7−	1.979	same as 1.984	..	..	..	..	..	<6.5	..	..
6−	1.9613	rot. state 1.810	7−6	..	..	..	..	>7	..	..
..	..	$624\uparrow+512\downarrow$ n	6−6	1.9	..	..	1 u	..	..	..
7−	1.9607	same as 1.984	..	..	..	..	..	6.1	..	..
6−	1.830	$624\uparrow+512\downarrow$ n	6−6	1.9	..	..	1 u	6.6	..	..
..	..	rot. state 1.290	6−2	..	..	..	1 K(1 u)	..	..	..
..	..	rot. state 1.554	6−4	..	..	..	1 K(1*h)	..	..	..
6−	1.811	same as 1.830	..	..	..	..	..	7.2	..	..
5−	1.810	$624\uparrow+510\uparrow$ n	5−5	1.5	..	..	1*h	>6.8	..	..
6+	1.757	$404\downarrow+402\uparrow$ p	6+6	1.4	..	..	ah	..	..	..
4−	1.554	$624\uparrow-510\uparrow$ n	4−4	1.5	ah	6.8	..	..	1*h	>7
2−	1.290	$514\uparrow-402\uparrow$ p	2−2	1.3	aF	8.2	..	..	1 u	6.3
2+	1.258	collective	2+0	..	1 K	≈ 9.7	..	..	..	..
2+	1.222	collective	2+2	..	1?	≈ 8.6	..	..	..	..

† 1) $404\downarrow-510\uparrow$ 2) $402\uparrow+624\uparrow$ 3) $402\uparrow-624\uparrow$

to far exceed that exciting it. Direct capture does not therefore seem ruled out by the experimental data.

It seems somewhat surprising that the $7-\ \Sigma=1$ state of the $514\uparrow\pm402\uparrow$ doublet does not occur lower in energy than ≈ 2 MeV, because with the present assignment the energy of spin splitting for this level is ≈ 700 keV. That it does occur at such a high excitation energy seems to be borne out by the absence of any strongly populated state around 1500 keV which decays directly to the 6+ and 8+ members of the ground-state band.

A = 184.

Re^{184} electron capture populates levels with $I\pi=3+$ and $2+$, and decays only weakly, if at all, to the $K\pi=0+$ ground state band in W^{184}, indicating $2\pm$, $3\pm$ as the most probable spin assignments. A $K\pi=3-$ configuration $402\uparrow+510\uparrow$ is predicted as the Re^{184} ground state, in good agreement with the experimental data.

${}^{108}_{74}W^{182}$

Table III 16b.

State	$K\pi$	Energy (MeV) Calc.	Energy (MeV) Exper.	${}^{109}_{73}Ta^{182}$ 3 − $p = K-1$; $n = K+1$ Class	log ft	${}^{107}_{75}Re^{182}$ 7 + $p = K+1$; $n = K$ Class	log ft	${}^{107}_{75}Re^{182}$ 2 + $p = K+1$; $n = K$ Class	log ft
neutron levels									
$K, K+1$	4 −	1.5	1.554	ah	6.8 (6.9)	..	..	1^*h	..
	5 −	..	1.810	..	..	1^*h	large	..	..
K, K; $K+1, K+1$	0 +	1.6	..	..	..	..	..	..	..
$K, K+2$	6 −	1.9	1.830	..	..	1 u	6.6 (7.3)	..	..
	3 −	..	2.024	aF	..	..	..	1 u	6.4 (7.3)
$K-1, K+1$	4 +	1.9	..	1 u	..	..	..	..	..
	3 +	..	..	$1A(1u)$	..	..	..	aF	..
$K-1, K$	8 −	2.0	..	..	..	1 h	..	..	..
	1 −	..	..	..	..	..	..	1 h	..
$K-2, K+1$	2 +	2.1	..	1 h	..	..	..	aF	..
	3 +	..	..	$1A(1h)$	..	..	..	aF	..
$K, K+3$	1 −	2.2	..	..	..	..	..	1 u	..
	8 −	..	..	..	..	1 u	..	..	..
$K+1, K+2$	2 +	2.2	..	$1A(1^*h)$	..	..	..	aF	..
	1 +	..	..	1^*h	..	..	..	aF	..
$K-2, K$	2 −	2.3	2.184	aF	..	..	..	1 u	(6.7)
	7 −	..	..	..	..	1 u	(6.7)	..	..

$K-2 = 512\uparrow$, $K-1 = 514\downarrow$, $K = 624\uparrow$, $K+1 = 510\uparrow$, $K+2 = 512\downarrow$, $K+3 = 503\uparrow$.

State	$K\pi$	Calc.	Exper.	Class	log ft	Class	log ft	Class	log ft
proton levels									
$K, K+1$	2 −	1.3	1.290	aF	8.2	..	..	1 u	6.3 (6.4)
	7 −	..	..	..	..	1 u	(6.4)	..	..
$K-1, K+1$	6 +	1.4	1.757	..	..	ah	(6.7)	..	..
	1 +	..	..	1^*h	..	..	..	ah	(6.5)
K, K; $K+1, K+1$	0 +	1.8	..	..	..	..	..	..	..
$K-1, K+2$	4 +	2.0	..	$1A(1u)$	..	..	..	..	..
	3 +	..	..	1 u	..	..	..	aF	..
$K-1, K$	8 −	2.0	..	..	..	1 F	..	..	..
	1 −	..	..	..	..	..	..	1 F	..
$K, K+2$	4 −	2.0	..	aF	..	..	..	..	..
	5 −	..	..	..	..	1^*F	..	..	..
$K-2, K+1$	3 +	2.1	..	1 F	..	..	..	$aA(4)$	..
	2 +	..	..	1 F	..	..	..	$a(4)$	..
$K+1, K+2$	2 +	2.2	..	1 F	..	..	..	$a(4)$	..
	3 +	..	..	1 F	..	..	..	$aA(4)$	..
$K-3, K+1$	1 −	2.5	..	..	..	..	..	$1u$	..
	6 −	..	..	..	..	1 u	..	..	..
$K+1, K+3$	4 +	2.6	..	1 F	..	..	..	..	..
	1 +	..	..	..	..	..	..	$a(2)$	..

$K-3 = 523\uparrow$, $K-2 = 411\downarrow$, $K-1 = 404\downarrow$, $K = 514\uparrow$, $K+1 = 402\uparrow$, $K+2 = 400\uparrow$, $K+3 = 402\downarrow$.

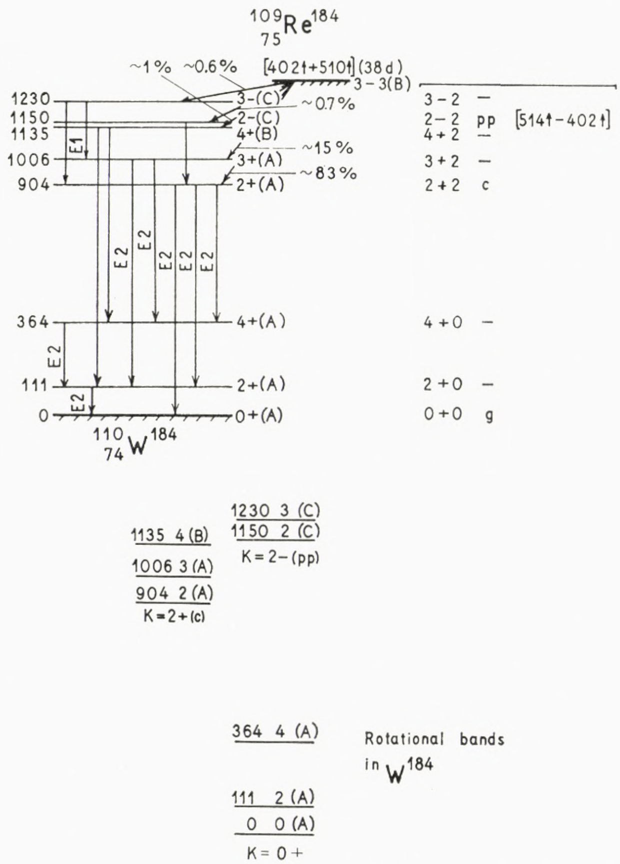


Fig. 19. A = 184.

The decay scheme of 38 *d* Re^{184} has been studied by C. J. Gallagher, Jr., D. Strominger, and J. P. Unik, Phys. Rev. **110**, 725 (1958) and HHM 61. The spins of the 904 and 1006 keV levels have been established by E. Bodenstedt, E. Matthias, H. J. Körner, E. Gerdau, F. Frisius, and D. Hovestadt, Nuclear Phys. **15**, 239 (1960), who also established the half-life of the isomer. Recently a second isomer of Re^{184} with a 165 *d* half-life has been reported by N. R. Johnson, Bull. Am. Phys. Soc. **6**, 73 (1961). Two levels at 1106 are 1101 keV proposed by HHM 61 have not been included in the figure.

Re^{184} (3−3)−1 Table III 17. $^{110}_{74}$W^{184}

Experimental		Theoretical			1†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft*
2−	1.150	514 ↑ − 402 ↑ p	2−2	1.3	a(4)	7.3 to 8.3
2+	.904	510 ↑ + 512 ↓ n	2+2	..	1 u	7.0 to 7.4
..	..	402 ↑ − 400 ↑ p	2+2	..	1A(1 u)	..
..	..	collective	2+2	..	1?	..

† 402 ↑ + 510 ↑
* if $1.33 \leq Q_{EC} \leq 1.6$

Whether the $K\pi = 2+$ band in W^{184} should be assigned as a collective or intrinsic band is somewhat uncertain, because the lowest-lying neutron excitation is the $K\pi = 2+$ state $510\uparrow + 512\downarrow$, which should be populated by a 1 u electron capture branch. The log ft estimated for the branch (≈ 7.0) is somewhat larger than that expected for a 1 u transition, but actually less than the transition rate for the same single-particle transition in W^{185} and W^{187} decay. The calculated energy of the state is considerably higher than the experimental energy.

Weak branches to the $2-$ proton level $514\uparrow - 402\uparrow$ may be observed, but the assignment of the 2– level at 1150 keV is uncertain.

A = 186.

Re^{186} is clearly stablished as the 1– configuration $402\uparrow - 512\downarrow$. The log $ft = 7.7$ for the beta branch to the Os^{186} ground state is somewhat larger than ordinarily ob-

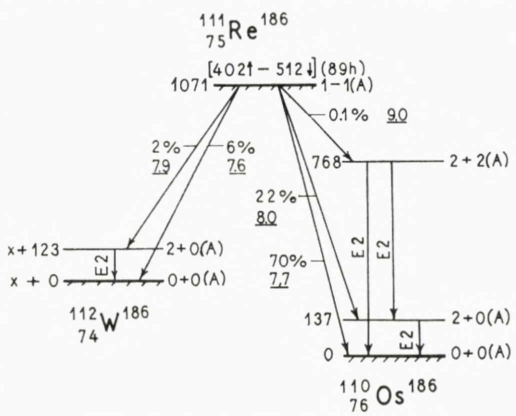


Fig. 20. A = 186.
The data on the decay of Re^{186} is as reported in SHS and the NDS. The electron capture decay energy is taken from a recent adjustment of mass values in the rare earth region (A. H. Wapstra, priv. comm., May 1961).

$Re^{186}(1-1)-1$ Table III 18. ${}^{102}_{74}W^{186}$

Experimental		Theoretical			1†	
$I\pi$	E	Final configuration	$I\pi K$	E	Class	log ft*
2+	.768	collective (Os^{186})	2+2	–	1?	9.0
2+	.137	ground (Os^{186})	2+0	–	1 u	8.0
0+	0	ground (Os^{186})	0+0	–	1 u	7.7
2+	.123	ground (W^{186})	2+0	–	1 u	7.9
0+	0	ground (W^{186})	0+0	–	1 u	7.6

† $402\uparrow - 512\downarrow$.
* if $Q_{EC} = .70$.

served for a 1 u transition, although the similar log ft = 7.5 observed for the odd-particle transition in the $W^{185} \rightarrow Re^{185}$ decay suggests that the deformation may be changing rapidly here. The energy of the 768 keV 2+ state in Os^{186} suggests a collective excitation. No energies for $N = 112$ or $Z = 76$ have been calculated (see Section II).

IV. DISCUSSION AND CONCLUSIONS

A. K Selection Rules

1. *Beta decay.* The K selection rules have been found to be very important in beta decay. While such classic examples as the $\nu = 6$ ($\nu = K_f - K_i - \lambda$, where λ is the multipole order of the transition) forbidden beta decay of the high spin isomer of Lu^{176} clearly demonstrate the validity of the rules, even $\nu = 1$ forbidden transitions are appreciably retarded. A forbiddenness of $\approx 10^2$ per unit of K forbiddenness seems to be generally observed.

2. *Gamma-ray decay.* Data on gamma-ray retardations are at present somewhat scarce. The available evidence indicates that appreciable retardations due to K-forbiddenness occur. Particularly striking examples are the Hf^{178} and Hf^{180} isomers, where the $\nu = 7$ forbidden transitions are retarded by factors of $\approx 10^{13}$ and $\approx 10^{15}$, respectively.

B. K Intensity Rules

1. *Beta decay.* In Table IV 1 are presented the available experimental data on relative ft values for beta decay from an odd-odd nucleus to rotational levels in the even-even ground state band. For Ho^{162}, Ho^{164}, Tm^{170}, Tm^{172}, Ta^{178}, Re^{186}, and Re^{188} the K quantum numbers are established by the absolute values of the transition rates which establish the configurations. In these cases (with the exception of Ho^{162}, in which the experimental branching ratio has a very large experimental uncertainty) the experimental branching ratios are within experimental error of the theoretical predictions. From these data we conclude that in general the K intensity rules are valid experimentally. The K quantum numbers of Lu^{176} and Ta^{180} have therefore been assigned on the basis of the observed ratios. No theoretical values are listed for the 0− states 9.3 h Eu^{152} and 2.7 h Ho^{166}, because in these cases the matrix elements for transitions to the 0+ and 2+ rotational states are clearly not identical. The case of Eu^{156} seems to be the single exception to the generally valid rules, but in this case the spin is not clearly established.

2. *Gamma-ray Decay.* We have made no systematic effort to classify the K quantum numbers of even-even levels on the basis of the K intensity rules, nor to investigate this question in any detail. However, K intensity rules for interband transitions have been checked in a systematic way for the decay of the $K = 2+$ [38] and $K = 0$

Table IV 1.

Initial nucleus	Initial $I\pi K$	Final nucleus	Decay fraction (%) to final state of $(I\pi)$			Total decay energy (MeV)	Total t 1/2	Ratio $\frac{ft(I_i \pi_i K_i \to 2+0)}{ft(I_i \pi_i K_i \to 0+0)} = \frac{\langle I_i L K_i K_f - K_i \mid 00 \rangle^2}{\langle I_i L K_i K_f - K_i \mid 20 \rangle^2}$		
			0+	2+	4+			Theor.	Exp.	Ref.
$^{89}_{63}\mathrm{Eu}^{152m}$	0 − 0	$^{90}_{62}\mathrm{Sm}^{152}$	.007	.004	–	1.917	9.3h	..$^{\alpha}$	0.8 ± 0.3	a
$^{95}_{67}\mathrm{Ho}^{162}$	1 + 1	$^{96}_{66}\mathrm{Dy}^{162}$	44	56	–	2.160	11.8m	2.0	0.9 ± 0.7	b
$^{97}_{67}\mathrm{Ho}^{164}$	1 + 1	$^{96}_{68}\mathrm{Er}^{164}$	35	14	–	0.99	$\gtrsim$25m	2.0	1.8 ± 0.5	c, d
$^{99}_{67}\mathrm{Ho}^{166}$	0 − 0	$^{98}_{68}\mathrm{Er}^{166}$	52	48	–	1.854	27h	..$^{\alpha}$	0.8 ± 0.35	e
$^{101}_{69}\mathrm{Tm}^{170}$	1 − 1	$^{100}_{70}\mathrm{Yb}^{170}$	76	24	–	0.969	129d	2.0	1.9 ± 0.2	f
$^{103}_{69}\mathrm{Tm}^{172}$	2 − 2	$^{102}_{70}\mathrm{Yb}^{172}$	23	41	1.2	1.920	63.6h	0.70:1:14$^{\beta}$	0.63:1:13$^{\beta}$	g
$^{105}_{71}\mathrm{Lu}^{176}$	1 − 0	$^{104}_{72}\mathrm{Hf}^{176}$	42	58	–	1.314	3.7h	0.5	0.56 ± 0.16	h
$^{105}_{73}\mathrm{Ta}^{178}$	1 + 1	$^{106}_{72}\mathrm{Hf}^{178}$	59	35	–	1.912	9.3m	2.0	1.6 ± 0.8	i
$^{107}_{73}\mathrm{Ta}^{180}$	1 + 1	$^{108}_{72}\mathrm{Hf}^{180}$	60	27	–	0.865	8.15h	2.0	2.0 ± 0.3	j, k
$^{107}_{73}\mathrm{Ta}^{180}$	1 + 1	$^{106}_{74}\mathrm{W}^{180}$	6.9	3.2	–	0.710	8.15h	2.0	1.8 ± 0.3	j
$^{111}_{75}\mathrm{Re}^{186}$	1 − 1	$^{112}_{74}\mathrm{W}^{186}$	6	2	–	0.700	89h	2.0	1.8 ± 0.5	e, k
$^{111}_{75}\mathrm{Re}^{186}$	1 − 1	$^{110}_{76}\mathrm{Os}^{186}$	70	22	–	1.071	89h	2.0	2.1 ± 0.2	l
$^{113}_{75}\mathrm{Re}^{188}$	1 − 1	$^{112}_{76}\mathrm{Os}^{188}$	73	24	–	2.116	17h	2.0	2.1 ± 0.2	m

Table IV. 1. *Comparison of theoretical and experimental relative reduced transition probabilities for beta decay of strongly deformed odd-odd nuclei.*

(a) D. Alburger, S. Ofer, and M. Goldhaber, Phys. Rev. **112**, 1998 (1958).
(b) M. Jørgensen, O. B. Nielsen, and O. Skilbreid, Nuclear Phys. **24**, 443 (1961).
(c) M. Jørgensen, O. B. Nielsen, and O. Skilbreid, Nuclear Phys. (to be published).
(d) H. N. Brown and R. A. Becker, Phys. Rev. **96**, 1372 (1954).
(e) R. L. Graham, J. L. Wolfson, and M. A. Clark, Phys. Rev. **98**, 1173A (1955).
(f) R. L. Graham, J. L. Wolfson, and R. E. Bell, Can. J. Phys. **30**, 459 (1952).
(g) P. G. Hansen, O. J. Jensen, and K. Wilsky, Nuclear Phys. **27**, 516 (1961).
(h) I. Řezánka, J. Frána, J. Adam, and L. K. Peker, Izvest. Akad. Nauk SSSR, Ser. Fiz., **26**, 127 (1961).
(i) C. J. Gallagher, Jr., H. L. Nielsen, and O. B. Nielsen, Phys. Rev. **123**, 1590 (1961).
(j) C. J. Gallagher, Jr., M. Jørgensen, and O. Skilbreid, Nuclear Phys. (to be published).
(k) A. H. Wapstra, priv. comm., May 1961.
(l) F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner, Jr., Phys. Rev. **103**, 921 (1956).
(m) K. O. Nielsen and O. B. Nielsen, Nuclear Phys. **5**, 319 (1958).
(α) Different matrix elements in transitions to 0+ and 2+ states.
(β) $ft(2-2\to 2+0):ft(2-2\to 0+0):ft(2-2\to 4+0)$.

bands(22) to the ground-state band. In order to understand the branchings for the $K = 2+$ bands it has been necessary to postulate appreciable band mixing(38). Branchings from the $K = 0-$ levels, however, (for the $A > 220$ mass region) have shown a remarkable consistency with theoretical prediction.(22) The branchings for the 963 keV level in Sm^{152} and the 1663 keV level in Er^{166} are also consistent with the $K = 0$ interpretation of these levels. In general the data on the decay of other types of levels are not sufficiently precise to test the predictions in detail. Similarly, data on interband transitions, which have been found to agree well with theory in the odd-mass nuclei, are scarce in even-mass nuclei and we therefore do not consider these questions here.

C. Log ft Values

In Tables IV 2a, b and c we summarize the log ft's for single-particle transitions between two quasi-particle states. The data are tabulated so that all cases of the same transition are listed together. The single-particle transition rate used to determine the magnitude of the matrix element is also listed. The data indicate that the transition rates lie within the same range in odd and even-mass nuclei if no additional selection

Table IV 2a

Transition	Initial state $I\pi K$	Initial state Configuration	Final state $I\pi K$	Final state Configuration	R	log $(ft)_e$	log $(ft)_c$	log $[(ft)_c R\eta]$
allowed unhindered								
$Ho^{167} \to Er^{167}$	7/2 − 7/2	523 ↑	5/2 − 5/2	523 ↓	0.52	*4.8*	4.8[(a)]	4.6
$Ho^{160} \to Dy^{160}$	5 + 5	523 ↑ + 521 ↑	4 + 4	523 ↓ + 521 ↑ (n)	0.36	4.8	4.9	4.4
$Ho^{162} \to Dy^{162}$	1 + 1	523 ↑ − 523 ↓	0 + 0	ground	0.25	4.7	5.3	3.9
$Ho^{162} \to Dy^{162}$	6 − 6	523 ↑ + 642 ↑	5 − 5	523 ↓ + 642 ↑ (n)	0.20	4.6	5.2	4.0
$Ho^{164} \to Dy^{164}$	1 + 1	523 ↑ − 523 ↓	0 + 0	ground	0.35	~ 5.3	5.3	4.3
$Ho^{164} \to Er^{164}$	1 + 1	523 ↑ − 523 ↓	0 + 0	ground	0.20	5.4	5.5	4.3
$Tm^{164} \to Er^{164}$	1 + 1	523 ↑ − 523 ↓	0 + 0	ground	0.61	≤ 5.0	5.1	4.2
$Yb^{164} \to Tm^{164}$	0 + 0	ground	1 + 1	523 ↑ − 523 ↓	0.08	≲ 5.0	5.5	3.8
$Dy^{166} \to Ho^{166}$	0 + 0	ground	1 + 1	523 ↑ − 523 ↓	0.44	4.9	5.0	4.5
$Ho^{166} \to Er^{166}$	0 − 0	523 ↑ − 633 ↑	1 − 1	523 ↓ − 633 ↑ (n)	0.38	5.2	5.1	4.8
$Ho^{166} \to Er^{166}$	7 − 7	523 ↑ + 633 ↑	6 − 6	523 ↓ + 633 ↑ (n)	0.38	≲ 6.7	5.1	..
$Yb^{175} \to Lu^{175}$	7/2 − 7/2	514 ↓	9/2 − 9/2	514 ↑	0.32	*4.7*	4.7[(b)]	4.2
$Ta^{178} \to Hf^{178}$	1 + 1	514 ↑ − 514 ↓	0 + 0	ground	0.22	4.6	5.4	3.6
$Ta^{178} \to Hf^{178}$	7 − 7	404 ↓ + 514 ↓	8 − 8	404 ↓ + 514 ↑ (p)	0.34	4.9	4.7	4.4
$W^{178} \to Ta^{178}$	0 + 0	ground	1 + 1	514 ↑ − 514 ↓	0.36	≤ 5.5	4.8	..

(a) See ref. 6.
(b) See NDS.

Table IV 2b

Transition	Initial state $I\pi K$	Initial state Configuration	Final State $I\pi K$	Final State Configuration	R	log $(ft)_e$	log $(ft)_c$	log $[(ft)_c R\eta]$
allowed hindered								
$W^{181} \to Ta^{181}$	9/2 + 9/2	624 ↑	7/2 + 7/2	404 ↓	0.34	*6.6*[(a)]	6.6[(a)]	6.2
$Ta^{178} \to Hf^{178}$	7 − 7	404 ↓ + 514 ↓	8 − 8	624 ↑ + 514 ↓ (n)	0.21	4.9	6.9	4.2
$Ta^{180} \to Hf^{180}$	1 + 1	404 ↓ − 624 ↑	0 + 0	ground	0.23	6.0	7.1	5.0
$Ta^{180} \to W^{180}$	1 + 1	404 ↓ − 624 ↑	0 + 0	ground	0.38	6.8	6.9	6.1
$Ta^{182} \to W^{182}$	3 − 3	404 ↓ − 510 ↑	4 − 4	624 ↑ − 510 ↑ (n)	0.22	6.9	6.9	6.2

(a) See NDS.

TABLE IV 2c.

Transition	Initial state		Final state		R	log $(ft)_e$	log $(ft)_c$	log $[(ft)_e R\eta]$
	$I\pi K$	Configuration	$I\pi K$	Configuration				
first forbidden unhindered								
$Dy^{165}\to Ho^{165}$	7/2+7/2	633 ↑	7/2−7/2	523 ↑	0.33	6.2(a)	6.2(a)	5.7
$Dy^{166}\to Ho^{166}$	0+0	ground	0−0	523 ↑ − 633 ↑	0.19	7.1(b)	6.4	6.4
$Ho^{166}\to Er^{166}$	0−0	523 ↑ − 633 ↑	0+0	ground	0.33	8.1(b)	6.1	7.7
$Tm^{168}\to Er^{168}$	3+3	411 ↓ − 633 ↑	3−3	411 ↓ − 523 ↑ (p)	0.30	6.0	6.2	5.5
$Tm^{167}\to Er^{167}$	1/2+1/2	411 ↓	1/2−1/2	521 ↓	0.37	6.5(c)	6.5(c)	6.1
$Tm^{168}\to Er^{168}$	3+3	411 ↓ − 633 ↑	3−3	521 ↓ − 633 ↑ (n)	0.36	7.7	6.5	7.2
$Tm^{172}\to Yb^{172}$	2−2	411 ↓ − 512 ↑	2+2	521 ↓ − 512 ↑ (n)	0.29	6.8	6.6	6.3
$Lu^{172}\to Yb^{172}$	4−4	404 ↓ + 521 ↓	4+4	404 ↓ + 411 ↓ (p)	0.38	6.1	6.5	5.3
$Ta^{175}\to Hf^{175}$	7/2+7/2	404 ↓	7/2−7/2	514 ↓	0.31	6.4(d)	6.4(d)	5.8
$Lu^{172}\to Yb^{172}$	4−4	404 ↓ + 521 ↓	4+4	514 ↓ + 521 ↓ (n)	0.40	6.5	6.3	6.1
$W^{181}\to Ta^{181}$	9/2+9/2	624 ↑	9/2−9/2	514 ↑	0.37	6.8(c)	6.8(c)	6.4
$Re^{182}\to W^{182}$	2+2	402 ↑ − 624 ↑	2−2	402 ↑ − 514 ↑ (p)	0.28	6.3	6.9	5.7
$W^{185}\to Re^{185}$	3/2−3/2	512 ↓	5/2+5/2	402 ↑	0.38	7.5(a)	7.5(a)	7.1
$Re^{186}\to W^{186}$	1−1	402 ↑ − 512 ↓	0+0	ground	0.23	7.6	8.2	6.5
$Re^{186}\to Os^{186}$	1−1	402 ↑ − 512 ↓	0+0	ground	0.38	7.7	7.9	6.8
$Re^{188}\to Os^{188}$	1−1	402 ↑ − 512 ↓	0+0	ground	0.30	8.0	8.1	7.0

(a) See ref. 6.

(b) Different operators are probably responsible for the transition in this case because of the $0-\to 0+$ transition.

(c) See NDS.

(d) B. HARMATZ, T. H. HANDLEY, and J. W. MIHELICH, Phys. Rev. **119**, 1345 (1960).

Table IV. 2. Comparison of beta decay transition rates for single-particle transitions between two-quasi-particle states. Log $(ft)_e$ is the experimental log ft reported in Section III. $R = R_Z R_N$ is the correction which accounts for the different distribution of paired-particle amplitudes in the proton and neutron cores in the final and initial states (i. e. the superfluid correction). The calculated log $(ft)_c$ is discussed in Section II. Log $[(ft)_e R\eta]$ is the single-particle transition rate from which the superfluid (R) and statistical (η) corrections have been excluded. a) allowed unhindered beta transitions; b) allowed hindered beta transitions; c) first forbidden unhindered beta transitions.

rules arising from the two-particle system apply. These results support the validity of the conclusion previously obtained from a less extensive classification.[8]

The range of rates applying to the various Alaga-selection rule-classifications in odd-mass nuclei is (cf. MOTTELSON and NILSSON)

$$4.5 \lesssim \log ft \lesssim 5.0 \quad au$$

$$6.0 \lesssim \log ft \lesssim 7.5 \quad ah$$

$$6.0 \lesssim \log ft \lesssim 7.5 \quad 1\,u$$

$$7.5 \lesssim \log ft \lesssim 8.5 \quad 1\,h.$$

The effect of excluding pairing correlation corrections from the single-particle transition probabilities tends to increase the transition rates and decrease the spread of the separate cases, so that after correction the range of rates for the various classes is

$$4.0 < \log[(ft)_e R\eta] < 4.7 \quad au$$
$$5.5 < \log[(ft)_e R\eta] < 6.5 \quad ah$$
$$5.5 < \log[(ft)_e R\eta] < 6.5 \quad 1\,u.$$

No data on 1 h transitions are included in this compilation because the available data are few. This investigation of the β-decay rates thus shows that at least to first order the concept of independent quasi-particles is correct. In addition, the correction terms for beta decay rates calculated on the basis of the pairing correlation calculations which take into account the differences in properties between odd and even-mass systems produce a greater consistency in the observed single-particle rates than is possible to achieve without them.

D. Level Energies

A comparison of the experimental data discussed in Section III with the calculated level spectra leads us to several conclusions. First of all, the qualitative agreement of the predicted and observed level spectra shows the general validity of the concept of two-quasi-particle excitations in deformed even-even nuclei.

A feature of these spectra that becomes apparent from the comparison is that the degeneracy of the $\Omega_1 \pm \Omega_2$ doublet is removed at least partly as a result of spin splitting, in analogy to deformed odd-odd nuclei, and the $\Sigma = 0$ state of the configuration appears lower, as expected. In the best established case (Er^{166}), the splitting of the doublet is only ≈ 40 keV; in other cases, if the postulated analyses are correct, the splitting is as large as ≈ 800 keV.

A particularly important experimental fact is that the energy of at least one member of the $(K, K+1)$ configuration is observed at energies considerably less than the formal gap. Table IV 3 summarizes the information on the $(K, K+1)$ class of levels. Where both the $\Sigma = 0$ and $\Sigma = 1$ states are known, both are listed. We have also included the level rating (A, B, C) in the table, as all the data are not equally reliable. In this table are listed three energies for each configuration: the formal gap $2C$, the degenerate doublet energy calculated assuming blocking, and the experimental energy. It can be clearly seen from the table that all experimental energies are within calculational error of the calculated energies. This would appear to be a conclusive proof for the existence of blocking, except for the fact that the forces which split the states of the $\Omega_1 \pm \Omega_2$ doublet are not clearly understood. A further complication is that the energy splittings of the doublets are not well known experimentally either. For these reasons it does not seem possible at present to decide definitely what the strength of the blocking is relative to other residual interactions. However, we can

TABLE IV 3.

Nucleus	System	$K\pi$	Classification	Gap 2 C (MeV)	Energy (MeV)	
					Calculated	Observed
W^{184}	proton	2 −	C	1.61	1.3	1.150
	neutron	2 +	A^a	1.97	1.8	.904
W^{182}	proton	2 −	A	1.61	1.3	1.290
	proton ($\Sigma = 1$) . . .	7 −	B	1.61	1.3	1.961
	neutron	4 −	A	1.89	1.5	1.554
	neutron ($\Sigma = 1$) . .	5 −	C	1.89	1.5	1.810
Hf^{180}	proton	8 −	A	1.66	1.0	1.142
Hf^{178}	proton	8 −	A	1.66	1.0	1.148
	neutron	8 −	B	1.85	1.5	1.480
Yb^{172}	proton	3 +	B	1.80	1.4	1.664
	proton ($\Sigma = 1$) . . .	4 +	B	1.80	1.4	2.075
	neutron	3 +	B	1.65	1.3	1.174
	neutron ($\Sigma = 1$) . .	2 +	B^a	1.65	1.3	1.468
Er^{168}	proton ($\Sigma = 1$) . . .	3 −	B	1.82	1.3	1.543
	neutron ($\Sigma = 1$) . .	3 −	A	1.64	1.1	1.095
Er^{166}	neutron	6 −	B	1.63	1.6	1.785
	neutron ($\Sigma = 1$) . .	1 −	A	1.63	1.6	1.826
Dy^{162}	neutron	5 −	A	1.83	1.3	1.485
Dy^{160}	proton	2 −	A^b	1.90	1.4	1.260
	neutron	4 +	A	. .	1.6	1.694
Gd^{156}	proton	4 +	A^b	2.0	1.45	1.511
	proton ($\Sigma = 1$) . . .	1 +	B	2.0	1.45	1.966
	neutron	1 −	A^b	2.0	1.5	1.240
	neutron ($\Sigma = 1$) . .	4 −	A^b	2.0	1.5	2.042

(a) May be collective state.
(b) Experimental data excellent but configuration assignment not definite.

Table IV. 3. Energy of (K, $K+1$) states in deformed even-even nuclei. Unless otherwise indicated the observed state is the $\Sigma = 0$ state of the (K, $K+1$) configuration. The classification of states is as in Section III. The gap, or correlation, energy 2 C is the energy of the formal gap. The calculated energy includes the effect of blocking.

conclude on the basis of the experimental data that, if an average spin splitting of $\leqslant 500$ keV is assumed and if there is no shift in the center of gravity of the configuration, blocking does exist.

E. Evidence for Collective Excitations

The classification of the intrinsic spectrum made possible by the present model provides us with means of deciding whether there are states which have properties clearly different from two-quasi-particle states. In this category are particularly the well-known 2+ states at ≈ 1 Mev which are known to occur systematically. The energies of these states are consistently less than those calculated for intrinsic 2+

excitations, thus providing further support for their classification as collective excitations. However, the variation in energy of these 2+ states (once the deformation has stabilized) as a function of mass number seems to follow to some extent the energies of the intrinsic 2+ levels. With the exception of the Sm-Gd region, 0+ states which unambiguously have energies less than the energies calculated for the pair excitations have *not* systematically been observed. Little can be said about the 1− states which have been interpreted as collective octupole excitations, except that the two 1− states that can be assigned in this category, in Sm^{152} and Er^{166}, both appear where intrinsic excitations of the same nature are expected, but at energies somewhat lower than calculated.

F. General Conclusions

The most important results that appear from the present analysis are that the intrinsic states of deformed even-even nuclei can be consistently described as two-quasi-particle excitations, and that beta decay selection rules based on two-quasi-particle wave functions can describe the beta decay rates observed. In general, the assignments are clearest where the data are most comprehensive. In addition, the general fit of the calculated excitation energies of the two-quasi-particle states with the experimental energies supports the inherent validity of the model and makes it appear to be a useful basis for further studies of other properties of the two-quasi-particle system.

In addition to the calculation of properties calculable within the framework of the model, there is already clearly a need for the introduction of a term to account for the splitting of the $\Omega_1 \pm \Omega_2$ doublets; also, the presence of certain exceptions to the expected behaviour (like that of the 8− states in Hf^{178}) indicates greater complexity than is accounted for by the model. From these apparent discrepancies it is thus clear that more experimental information is needed before more detailed conclusions can be drawn about the strength of residual quasi-particle interactions.

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References

1. M. G. MAYER, Phys. Rev. **78**, 16 (1950).
2. O. HAXEL, J. H. D. JENSEN, and H. E. SUESS, Ergeb. Exakt. Naturw. **26**, 244 (1952).
3. A. BOHR, Mat. Fys. Medd. Dan. Vid. Selsk. **26**, No. 14 (1952).
4. A. BOHR and B. R. MOTTELSON, Mat. Fys. Medd. Dan. Vid. Selsk. **27**, No. 16 (1953).
5. S. G. NILSSON, Mat. Fys. Medd. Dan. Vid. Selsk. **29**, No. 16 (1955).
6. B. R. MOTTELSON and S. G. NILSSON, Mat. Fys. Skr. Dan. Vid. Selsk. **1**, No. 8 (1959).
7. C. J. GALLAGHER, Jr. and S. A. MOSZKOWSKI, Phys. Rev. **111**, 1282 (1958).
8. C. J. GALLAGHER, Jr., Nuclear Phys. **16**, 215 (1960).
9. A. BOHR, B. R. MOTTELSON, and D. PINES, Phys. Rev. **110**, 936 (1958).
10. S. T. BELYAEV, Mat. Fys. Medd. Dan. Vid. Selsk. **31**, No. 11 (1959).
11. V. G. SOLOVIEV, JETP SSSR **35**, 823 (1958); **36**, 1869 (1959); Nuclear Phys. **9**, 655 (1958/59).
12. V. G. SOLOVIEV, Doklady Akad. Nauk. SSSR **133**, 325 (1960).
13. K. ALDER, A. BOHR, T. HUUS, B. MOTTELSON, and A. WINTHER, Revs. Modern Phys. **28**, 432 (1956).
14. R. K. SHELINE, Revs. Modern Phys. **32**, 1 (1960).
15. D. R. BÈS, Mat. Fys. Medd. Dan. Vid. Selsk. **33**, No. 2 (1961).
16. J. GRIFFIN and M. RICH, Phys. Rev. **118**, 850 (1960).
17. S. G. NILSSON and O. PRIOR, Mat. Fys. Medd. Dan. Vid. Selsk. **32**, No. 16 (1960).
18. V. G. SOLOVIEV, Mat. Fys. Skr. Dan. Vid. Selsk. **1**, No. 11 (1961).
19. V. G. SOLOVIEV, Doklady Akad. Nauk. SSSR **137**, 1350 (1961).
20. V. G. SOLOVIEV, JETP SSSR **40**, 654 (1961).

21. W. M. Hooke, Phys. Rev. **115**, 453 (1959).
22. F. S. Stephens, F. Asaro, and I. Perlman, Phys. Rev. **96**, 1568 (1954).
23. F. Asaro, I. Perlman, J. O. Rasmussen, and S. G. Thompson, Phys. Rev. **120**, 934 (1960).
24. D. A. Varshalovich and L. K. Peker, Izvest. Akad. Nauk SSSR **25**, 287 (1961).
25. D. Kurath, Argonne National Laboratory Report ANL-6130 and ANL-6214 (1960) (unpublished); N. D. Newby, Univ. of California Radiation Laboratory Report UCRL-9764 (July 1961) (unpublished).
26. G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. **29**, No. 9 (1955).
27. G. Alaga, Phys. Rev. **100**, 432 (1955); Nuclear Phys. **4**, 625 (1957).
28. D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).
29. G. Ewan, J. S. Geiger, and R. L. Graham (to be published).
30. C. E. Johnson, J. F. Schooley, and D. A. Shirley, Phys. Rev. **120**, 2108 (1960).
31. M. Jørgensen, O. B. Nielsen, and O. Skilbreid, Nuclear Phys. **24**, 443 (1961).
32. M. Jørgensen, O. B. Nielsen, and O. Skilbreid, Nuclear Phys. (to be published).
33. P. G. Hansen, K. Wilsky, D. J. Horen, and Lung-Wen Chiao, Nuclear Phys. **24**, 519 (1961).
34. C. J. Gallagher, Jr., O. B. Nielsen, O. Skilbreid, and A. W. Sunyar, Phys. Rev. (to be published).
35. B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **123**, 1758 (1961).
36. R. G. Helmer and S. B. Burson, Argonne National Laboratory Report ANL-6270 (January 1961) (unpublished); Phys. Rev. **123**, 978 (1961).
37. C. J. Gallagher, Jr. and H. L. Nielsen, Phys. Rev. (to be published).
38. O. B. Nielsen, *Proceedings of the Rutherford Jubilee International Conference* (Heywood and Co. Ltd., London) (in print).

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